Tube-Based Discrete Controller Design for Vehicle Platoons Subject to Disturbances and Saturation Constraints

Shuo Feng, Haowei Sun, Yi Zhang, *Member, IEEE*, Jianfeng Zheng, Henry X. Liu, *Member, IEEE*, and Li Li, *Fellow, IEEE*

***Abstract*— Cooperative adaptive cruise control (CACC) is a promising intelligent vehicle technology for improving traffic flow stability, throughput, and safety. One major control objective of CACC is to guarantee $\mathcal{L}_p$ string stability, i.e., $\mathcal{L}_p$-norm measured disturbance is uniformly bounded along the vehicle string. Most existing methods for string stability are laborious for implementation without considering either heterogeneous disturbances (e.g., tracking errors and unmodeled dynamics) or saturation constraints (e.g., input saturation). The decentralized model predictive control (MPC) method, which is a widely used feedforward control for string stability, suffers the burdens of computation cost and intervehicular communication. To fill these gaps, we distinguish different types of disturbances and use different ways to handle them. We use feedforward control for large yet infrequent disturbances and feedback control for small yet frequent disturbances. Different from MPC, our feedforward control is event-triggered so that the intervehicle communication and planning costs can be significantly reduced. Different from pure robust feedback control, our combination of feedback and feedforward control could reduce the conservation of the controller. Theoretical analysis and simulations show that the proposed method guarantees $\mathcal{L}_p$ string stability of vehicle platoons considering heterogeneous disturbances and saturation constraints.**

***Index Terms*— Cooperative adaptive cruise control (CACC), disturbance $\mathcal{L}_p$ string stability (DSS), feedback control, feedforward control, model predictive control, tube.**

I. Introduction

COOPERATIVE adaptive cruise control (CACC) is one of the promising intelligent vehicles technologies that contribute to improving traffic flow stability, throughput, and safety [1], [2]. Through the vehicle-to-vehicle and vehicle-to-infrastructure wireless communication, CACC systems can utilize much more information (e.g., accelerations of all vehicles) between vehicles to improve the performance of platoon control [3], [4].

Manuscript received September 5, 2018; revised November 7, 2018; accepted January 21, 2019. Date of publication February 18, 2019; date of current version April 13, 2020. Manuscript received in final form January 27, 2019. This work was supported in part by the National Natural Science Foundation of China under Grant 61790565, Grant 61603005, and Grant 71671100, in part by the Beijing Municipal Science and Technology Program under Grant D15110900280000, and in part by the Beijing Municipal Commission of Transport Program under Grant ZC179074Z. Recommended by Associate Editor N. Quijano. *(Corresponding author: Li Li.)*

S. Feng and H. Sun are with the Department of Automation, Tsinghua University, Beijing 100084, China (e-mail: s-feng14@mails.tsinghua.edu.cn; sun-hw15@mails.tsinghua.edu.cn).

Y. Zhang and L. Li are with the Department of Automation, Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing 100084, China (e-mail: zhyi@mail.tsinghua.edu.cn; li-li@mail.tsinghua.edu.cn).

J. Zheng and H. X. Liu are with the Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: zhengjf@umich.edu; henryliu@umich.edu).

Color versions of one or more of the figures in this article are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCST.2019.2896539

One major control objective of CACC is to guarantee the string stability. String stability refers to the uniform boundedness of all the states of the vehicle platoon independent of the platoon size if the initial states are all uniformly bounded [5]. This property ensures that any position disturbance of the leading vehicle will not result in amplified fluctuations to the following vehicle's position [6]. If the fluctuations are quantified by $\mathcal{L}_p$-norm, the string stability is called $\mathcal{L}_p$ string stability [7], [8].

Most existing methods for string stability design a feedback control with proper parameters, but cannot handle saturation constraints. Three types of methods are proposed for determining parameters: the first type tunes the parameters by transfer function analysis after designing the controller structure [3], [9], [10], the second type designs both the parameters and structures simultaneously by $\mathcal{H}_\infty$ control [7], [11], [12], and the third type determines the parameters by Lyapunov technologies [5], [13]–[16]. The common limitation of these methods is the difficulty of handling saturation constraints, e.g., input saturation and safety constraints, because of the conservation of the controller.

To solve the saturation constraints, a decentralized model predictive control (MPC)-like method (i.e., a feedforward control) is proposed to guarantee $\mathcal{L}_p$ string stability [17]. MPC can optimize future system behavior and utilize the first input in the optimal sequence at each control interval [18]–[21]. With additional constraints of the optimization problem, the proposed method [17] can guarantee $\mathcal{L}_p$ string stability of vehicle platoons.

The primary limitation for the decentralized MPC-like method is the heavy burden of computation and intervehicular communication to handle disturbances [18], [22], [23]. For each control interval (e.g., 0.1 s), the method is required to solve an optimization problem based on the updated information by intervehicular communication to handle possible disturbances.

To solve the above-mentioned problems, we propose an efficient robust $\mathcal{L}_p$ string stability control method with the following objectives.

1) The method guarantees the $\mathcal{L}_p$ string stability of vehicle platoons, handling disturbances and satisfying saturation constraints.
2) The method is much more efficient than the MPC-like method regarding intervehicular communication and computation.

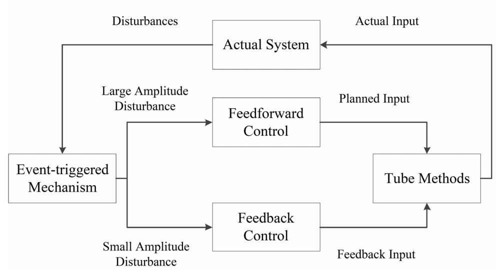

Fig. 1. Flowchart of the proposed method.

To this end, an integrated control policy is proposed, as shown in Fig. 1.

First, the disturbances are classified into two kinds by their amplitudes, i.e., initial tracking errors (large amplitude) and unmodeled dynamics (small amplitude). The initial tracking errors usually come from traffic behaviors, e.g., cut-in, lane-change, and braking behaviors, which have larger amplitude and lower frequency than the unmodeled dynamics.

Second, large amplitude disturbances trigger a feedforward control and intervehicular communication, while small amplitude disturbances trigger a feedback control. The feedforward control is powerful but costly concerning computation and intervehicular communication, whereas the feedback control is efficient but limited for saturation constraints. The proposed integrated method is both powerful and efficient with a proper balance.

Third, the feedforward control and feedback control are integrated based on tube methods to satisfy the saturation constraints. The conception of the tube method is first proposed by Mayne and Langson [24] and Langson *et al.* [25]. A minimal robust positively invariant (mRPI) [26] set is used to measure the error boundedness between the actual system and nominal system. The feedforward control is conducted for the nominal system in the tight sets, while the feedback control is conducted in the mRPI set. By the tube theorems [25], the controlled systems can satisfy saturation constraints.

In conclusion, we make the following contributions. We distinguish different types of disturbances and use different ways to handle them. We use feedforward control for large yet infrequent disturbances and feedback control for small yet frequent disturbances. Different from MPC which is a widely used feedforward control for string stability, our feedforward control is event-triggered so that the intervehicle communication and planning costs can be significantly reduced. Different from pure robust feedback control, our combination of feedback and feedforward control could reduce the conservation of the controller. Theoretical analysis and simulations show that the proposed control method achieves the two objectives mentioned above.

The rest of this brief is organized as follows. The problem formulation is proposed in Section II. In Section III, we propose the integrated control method. The control objectives and efficiency are analyzed in Section IV. Finally, we provide simulation results in Section V and conclusions in Section VI.

II. Problem Formulation

A. Notations and Scenario

The field of real number is denoted by $\mathbb{R}$, whereas $\mathbb{N} = \{1, 2, \dots\}$. For a vector $x \in \mathbb{R}^n$, its p-norm is given as $\|x\|_p = \left(\sum |x_i|^p\right)^{1/p}$, for $1 \le p < \infty$, and $\|x\|_\infty = \max_i |x_i|$. Given a Lebesgue measurable signal $x : I \to \mathbb{R}^n$, $\|x\|_{\mathcal{L}_p}$ denotes its $\mathcal{L}_p$ norm defined as $\|x\|_{\mathcal{L}_p} = \left(\int_I \|x\|_p^p \, dt\right)^{1/p} < \infty$, for $1 \le p < \infty$, and $\|x\|_{\mathcal{L}_\infty} = \operatorname{ess\,sup}_{t\in I} |x(t)|$. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. We recall Minkowski sum for sets $\mathbb{A}, \mathbb{B}$ is $\mathbb{A} \oplus \mathbb{B} = \{x + y | x \in \mathbb{A}, y \in \mathbb{B}\}$ and the Pontryagin difference is $\mathbb{A} \ominus \mathbb{B} = \{x | x + y \in \mathbb{A}, y \in \mathbb{B}\}$. The nominal value of a variable is denoted by its overline format, e.g., $\overline{x}$ denotes the nominal value of x. The disturbance value of a variable is denoted by its tilde format, e.g., $\tilde{x} = x - \overline{x}$. Notations are shown in detail in Table I.

TABLE I

Notations of the Variables in This Brief

Variables	Notations
S_m	$S_m = \{0\} \cup \{i \in \mathbb{N} \| 1 \le i \le m\}$.
$S_{e,m}$	$S_{e,m} = \{i \in \mathbb{N} \| 1 \le i \le m\}$.
s_i, v_i, a_i, u_i	The position, velocity, acceleration and input of the vehicle i.
$\bar{s}_i, \bar{v}_i, \bar{a}_i, \bar{u}_i$	The nominal values of the s_i, v_i, a_i, u_i.
$\bar{s}_i^a, \bar{v}_i^a, \bar{a}_i^a, \bar{u}_i^a$	The planned nominal position, velocity, acceleration and input of vehicle i.
$\tilde{s}_i, \tilde{v}_i, \tilde{a}_i, \tilde{u}_i$	The disturbance values of the s_i, v_i, a_i, u_i.
$x_i, \overline{x}_i, \tilde{x}_i$	The actual, nominal and disturbance values of the system state.
d_i	The standstill distance between vehicle i and $i-1$.
d_{min}	The minimal range error for safety requirement.
e_{si}, e_{vi}, e_{ai}	The range, velocity and acceleration errors between vehicle i and $i-1$.
$\bar{e}_{si}, \bar{e}_{vi}, \bar{e}_{ai}$	The nominal values of e_{si}, e_{vi}, e_{ai}.
$\tilde{e}_{si}, \tilde{e}_{vi}, \tilde{e}_{ai}$	The disturbance values of e_{si}, e_{vi}, e_{ai}.
τ	The sampling control time interval.
N_p	The total control interval numbers of the whole procedure.
ω_i	The new disturbances during the last time interval.
$\mathbb{Z}$	The minimal Robust Positively Invariant set (mRPI).
$\mathbb{W}$	The set of the new disturbances.
$\mathbb{X}_i, \mathbb{U}_i$	The state and acceleration constraints sets of actual vehicle i system.
$\overline{\mathbb{X}}, \overline{\mathbb{U}}$	The state and acceleration constraints sets of nominal vehicle i system.
$\mathbb{S}_i, \mathbb{V}_i$	The position and velocity constraints sets of actual system.
$\overline{\mathbb{X}}_{i,f}, \overline{\mathbb{U}}_{i,f}$	The terminal state and acceleration constraints sets of nominal vehicle i system.
$\overline{\mathbb{D}}_i$	The state constraints set from $\mathcal{L}_p$ string stability of nominal vehicle i system .

A vehicle platoon is a group of two or more closely spaced heterogeneous vehicles traveling with the same velocity in the same lane. As shown in Fig. 2, s_i denotes the position of the front bumper of the ith vehicle and L_i is the length of the ith vehicle. The leading vehicle is defined as the 0th vehicle. Let $S_m = \{0\} \cup \{i \in \mathbb{N} | 1 \le i \le m\}$ represent the set of all vehicles in a platoon of length $m + 1 \in \mathbb{N}$. Let $S_{e,m} = \{i \in \mathbb{N} | 1 \le i \le m\}$.

To define the scenario in this brief, we have the following assumptions.

Assumption 1: All vehicles in the platoon are equipped with CACC technologies.

Assumption 2: Each vehicle follows its predecessor vehicle at a constant time headway.

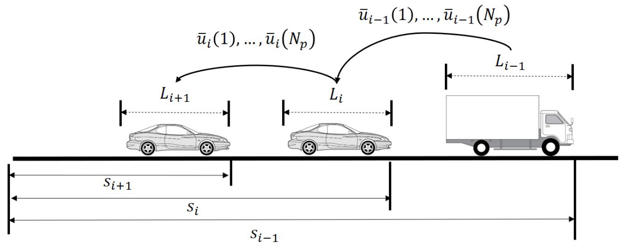

Fig. 2. Illustration of the scenario studied in this brief.

Assumption 3: During each time interval, the new unmodeled dynamics for each vehicle are bounded.

B. System Models

In many literatures, the dynamic model of vehicle is described by Newton's second law approximately as

$$m_i\dot{v}_i = m_i\xi_i - K_{di}\dot{x}_i^2 - d_{mi},$$
$$\dot{\xi}_i = -\frac{\xi_i}{\tau_i(\dot{x}_i)} + \frac{\eta_i}{m_i\tau_i(\dot{x}_i)} \tag{1}$$

where $m_i\xi_i$ represents the engine force applied to the vehicle, $K_{di}\dot{x}_i^2$ specifies the force generated by air resistance, and d_{mi} denotes the mechanical drag, $\tau(\dot{x}_i)$ is the vehicle engine time constant at velocity $\dot{x}_i$, m_i is the vehicle's mass, η_i represents the throttle input to the vehicle engine, $i \in S_m$.

The exact feedback linearization technique is used to convert the nonlinear model into a linear one for controller design. The same technique has been widely used before, e.g., [27]–[29], and [16]. If the following control law is adapted as:

$$\eta_i = m_iu_i + K_{di}\dot{x}_i^2 + d_{mi} + 2\tau_i K K_{di}\dot{x}_i\ddot{x}_i \tag{2}$$

the closed loop dynamics of each vehicle can be obtained as

$$\tau_i\dot{a}_i(t) + a_i(t) = u_i(t) \tag{3}$$

where $a_i(t) = \dot{v}_i(t)$ denotes the acceleration of vehicle i. For platoon control, the discrete state-space equation can be formulated by the continuous-time state-space equation of the system with zero-order hold method [30] and Taylor equation. By sampling the time interval τ, we obtain the discrete state-space model as

$$x_i(k+1) = A_ix_i(k) + B_iu_i(k) \tag{4}$$

where

$$x_i = \begin{bmatrix} s_i \\ v_i \\ a_i \end{bmatrix},\quad A_i = \begin{bmatrix} 1 & \tau & 0.5\tau^2 \\ 0 & 1 & \tau \\ 0 & 0 & 1-\frac{\tau}{\tau_i} \end{bmatrix},\quad B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{\tau}{\tau_i} \end{bmatrix} \tag{5}$$

and $k = 0, \ldots, N_p$.

If we denote the nominal value of a variable by its overline format, e.g., $\overline{x}$ denotes the nominal value of x, and

$$x_i(k) = \tilde{x}_i(k) + \bar{x}_i(k)$$
$$u_i(k) = \tilde{u}_i(k) + \overline{u}_i(k) \tag{6}$$

then we obtain the dynamics of the nominal, disturbance, and actual systems as

$$\bar{x}_i(k+1) = A_i\bar{x}_i(k) + B_i\bar{u}_i(k) \tag{7}$$
$$\tilde{x}_i(k+1) = A_i\tilde{x}_i(k) + B_i\tilde{u}_i(k) + \omega_i(k) \tag{8}$$
$$x_i(k+1) = A_ix_i(k) + B_iu_i(k) + \omega_i(k) \tag{9}$$

where $\omega_i(k) \in \mathbb{W} = \{\omega_i(k) \in \mathbb{R}^3 : \|\omega_i(k)\|_\infty \le c_w\}$ denotes the bounded unmodeled dynamics.

Next, we denote the errors of the nominal, disturbance, and actual systems. The errors of the actual system are denoted as

$$e_i = [e_{si}, e_{vi}, e_{ai}]^T$$
$$e_{si}(k) = s_{i-1}(k) - s_i(k) - h_i \cdot v_i(k) - d_i$$
$$e_{vi}(k) = v_{i-1}(k) - v_i(k)$$
$$e_{ai}(k) = a_{i-1}(k) - a_i(k) \tag{10}$$

where h_i refers to the time headway and d_i is a constant distance calculated by the standstill distance and the vehicle length. The nominal and disturbance values are denoted as $\bar{e}_i$ and $\tilde{e}_i$, respectively.

Final, we obtain the saturation constraints including the position, speed, acceleration, and input constraints. The constraint sets of the actual system can be denoted as

$$s_i \in \mathbb{S}_i = \{s_i \in \mathbb{R} : -d_{\min} \le e_{si}\}$$
$$v_i \in \mathbb{V}_i = \{v_i \in \mathbb{R} : v_{\min} \le v_i \le v_{\max}\}$$
$$a_i \in \mathbb{A}_i = \{a_i \in \mathbb{R} : -a_{\max} \le a_i \le a_{\max}\}$$
$$x_i \in \mathbb{X}_i = \{x_i \in \mathbb{R}^3 : v_i \in \mathbb{V}_i, s_i \in \mathbb{S}_i, a_i \in \mathbb{A}_i\}$$
$$u_i \in \mathbb{U}_i = \{a_i \in \mathbb{R} : -u_{\max} \le u_i \le u_{\max}\} \tag{11}$$

where $-d_{\min}$ represents the minimal range error for safety requirement.

C. Control Objectives

First, we recall the definitions of individual stability, $\mathcal{L}_p$ string stability, and disturbance $\mathcal{L}_p$ string stability (DSS).

Definition 1 (Individual Stability [8]): Consider the nominal system (7) and the error of the nominal system $\overline{e}_i$ (10). The nominal system is said to be individual stable if

$$u_0(t) = 0,\quad \forall t \ge 0 \to \lim_{t\to\infty} \overline{e}_i(t) = 0,\ i \in S_{e,m} \tag{12}$$

where u_0 is the input variable of the lead vehicle.

Definition 2 ($\mathcal{L}_p$ String Stability [8]): Consider the nominal system (7) and the error of the nominal system $\overline{e}_i$ (10). The system is said to be $\mathcal{L}_p$ string stable if there exist class $\mathcal{K}$ function α and constant $c > 0$ such that for any initial disturbance satisfying

$$|\bar{e}_{s1}(0)| < c \tag{13}$$

the solution $\overline{e}_{si}(t), i \in S_{e,m}$, exists for all $t > 0$ and satisfies

$$\|\overline{e}_{si}(t)\|_{\mathcal{L}_p} \le \alpha\left(|\bar{e}_{s1}(0)|\right). \tag{14}$$

Definition 3 (DSS [14]): Considering the actual system (9) and the error of the actual system e_i (10), the system is said to be disturbance $\mathcal{L}_p$ string stable if there exist class $\mathcal{K}$ function α

and constants $c > 0, c_w > 0, k_w > 0$ such that for any initial disturbance $e_{s1}(0)$ and new disturbance $w_i, i \in S_{e,m}$, satisfying

$$|e_{s1}(0)| < c, \|w_i\|_{\mathcal{L}_\infty} < c_w \tag{15}$$

the solution $e_{si}(t), i \in S_{e,m}$, exists for all $t > 0$ and satisfies

$$\|e_{si}(t)\|_{\mathcal{L}_p} \leq \alpha\left(|e_{s1}(0)|\right) + k_w c_w. \tag{16}$$

Remark 1: Compared with the definition of $\mathcal{L}_p$ string stability, the boundedness in DSS is composed of not only the initial disturbance but also the new disturbances. Because the α and k_w are independent of the platoon size, the spacing errors of the system have a uniform boundedness, $\alpha\left(|e_{s1}(0)|\right) + k_w c_w$, independent of platoon size.

Then, we propose the control objectives as following.

1) The nominal platoon system is individual stable.
2) The nominal platoon system is $\mathcal{L}_p$ string stable.
3) The actual platoon system is disturbance $\mathcal{L}_p$ string stable satisfying the saturation constraints.

III. Integrated Control Method

In this section, we propose the integrated control method based on tube methods, combining the feedforward control and feedback control.

A. Feedback Control

In this brief, a feedback control $\tilde{u}_i(k) = K_i \cdot \tilde{x}_i(k)$ is adapted for small-amplitude disturbances. The disturbance evolution (8) can be obtained as

$$\begin{aligned}\tilde{x}_i(k+1) &= (A_i + B_i \cdot K)\tilde{x}_i(k) + \omega_i(k),\\ &= A_{Ki}\tilde{x}_i(k) + \omega_i(k), \quad i \in S_{e,m}\end{aligned} \tag{17}$$

where $A_{Ki} = A_i + B_i \cdot K_i$.

The feedback control can be formulated as a discrete linear quadratic regulator (LQR) problem and solved as Problem 1:

Problem 1:

$$\min_{K_i} J_i = \sum_0^\infty \{\|Q\tilde{x}_{si}(k)\|_2 + \|L\tilde{x}_{vi}(k)\|_2 + \|R\tilde{u}_i(k)\|_2\} \tag{18}$$

subject to (8), where the weighting matrices, i.e., Q, L, and R, are symmetric and positive definite. [31]

B. Tube Methods

First, we recall the following well-known definitions [32].

Definition 4 (RPI Set): The set $\mathbb{Z} \subset \mathbb{R}^n$ is a robust positively invariant (RPI) set of the system (17) if $A_{Ki}\tilde{x}_i + \omega_i \in \mathbb{Z}$ for all $\tilde{x}_i \in \mathbb{Z}$ and all $\omega_i \in \mathbb{W}$, i.e., if and only if $A_{Ki}\mathbb{Z} \oplus \mathbb{W} \subset \mathbb{Z}$.

Definition 5 (Minimal RPI Set): The mRPI set $\mathbb{Z}$ of system (17) is the RPI set in $\mathbb{R}^n$ that is contained in every closed RPI set of system (17).

Next, we introduce the ϵ-approximation method [26] to compute the mRPI set $\mathbb{Z}$, where $\mathbb{Z} = \lim_{s\to\infty} F_s$ and

$$F_s = \bigoplus_{i=0}^{s-1} A_K^i \mathbb{W}, \quad F_0 = \{0\}. \tag{19}$$

Remark 2: According to (19), the mRPI set is independent of the platoon size. It is important for the proof of DSS.

Then, we define the tight sets and propose an assumption of existence. The constraint sets of actual system are defined

$$x_i(k) \in \mathbb{X}_i, \quad u_i(k) \in \mathbb{U}_i. \tag{20}$$

Let $\mathbb{Z}$ denote the mRPI set for disturbance system (17), then we obtain the tight sets as

$$\overline{\mathbb{X}}_i := \mathbb{X}_i \ominus \mathbb{Z}, \quad \overline{\mathbb{U}}_i := \mathbb{U}_i \ominus K_i\mathbb{Z}. \tag{21}$$

Assumption 4: We assume that the mRPI set, i.e., $\mathbb{Z}$, is sufficiently small such that all tight sets exist. [33]

Remark 3: This assumption is not unusual and represents a mild condition for many practical constraints and disturbances.

As proved by the tube methods [24], [25], if a new controller is conducted in the tight sets, the integrated controller of the feedback and the new controllers can control the actual system satisfying the original sets. A feedforward controller is designed as the new controller in this brief.

C. Feedforward Control

A feedforward controller is designed to handle large amplitude disturbances based on a convex optimization problem, satisfying the tight sets obtained by the tube methods. The control objectives of the feedforward control are making the nominal system to be individual stable and $\mathcal{L}_p$ string stable.

First, the terminal constraints are formulated to guarantee the individual stability of the nominal system. Let $\overline{s}_i^a(k)$, $\overline{v}_i^a(k)$, $\overline{a}_i^a(k)$ and $\overline{u}_i^a(k)$ represent the planned nominal position, velocity, acceleration, and input of vehicle i. The states of the nominal system can be predicted by the planned states as

$$\bar{s}_i(k) = \bar{s}_i^a(k), \quad \bar{v}_i(k) = \bar{v}_i^a(k), \ \bar{a}_i(k) = \bar{a}_i^a(k). \tag{22}$$

The terminal constraints are obtained as

$$\begin{aligned}\overline{x}_i \in \overline{\mathbb{X}}_{i,f} &= \{\overline{x}_i \in \mathbb{R}^3 : \overline{e}_i(N_p) = 0\}\\ \overline{u}_i \in \overline{\mathbb{U}}_{i,f} &= \{\overline{u}_i \in \mathbb{R} : \overline{u}_i(N_p) = 0\}.\end{aligned} \tag{23}$$

Next, according to Definition 2, the $\mathcal{L}_p$ string stability constraints are formulated as

$$\overline{x}_i(k) \in \overline{\mathbb{D}}_i = \{\overline{x}_i \in \mathbb{R}^3 : \|\overline{e}_{si}\|_{\mathcal{L}_p} \leq \alpha\left(|\overline{e}_{s1}(0)|\right)\}. \tag{24}$$

Third, we use a multiobjectives function to improve the performance of the platoon as

$$J_i = \sum_0^{N_p}\{\|Q_i\overline{e}_{si}(k)\|_2 + \|L_i\overline{e}_{vi}(k)\|_2 + \|R_i\overline{e}_{ai}(k)\|_2 + \|F_i\overline{u}_i(k)\|_2\} \tag{25}$$

where all weighting matrices, i.e., $Q_i, L_i, R_i, and F_i$, are symmetric and positive.

Final, the optimization problem of feedforward control is obtained as

Problem 2:

$$\min_{\overline{u}_i(0),\ldots,\overline{u}_i(N_p)} J_i \tag{26}$$

subject to

$$\overline{x}_i(k) \in \overline{\mathbb{X}}_i \cap \overline{\mathbb{X}}_{i,f} \cap \overline{\mathbb{D}}_i$$
$$\overline{u}_i(k) \in \overline{\mathbb{U}}_i \cap \overline{\mathbb{U}}_{i,f}, \tag{27}$$

where $k = 1, \ldots, N_p$. N_p denotes the predictive steps and is appropriate (e.g., large enough) to make Problem 2 feasible.

D. Integration Mechanism

The feedback control is conducted at each control interval, while the feedforward control and intervehicular communication are activated by an event-triggered mechanism.

If the disturbances of vehicle i exceed the boundedness, i.e., $\tilde{x}_i \notin \mathbb{W}$, the feedforward control of vehicle i will be triggered to generate a series of input trajectories, i.e., $\bar{u}_i(1), \ldots, \bar{u}_i(N_p)$, by solving Problem 2. The predictive states, i.e., $\bar{x}_i^a(1), \ldots, \bar{x}_i^a(N_p)$, and the input trajectories are transmitted to the following vehicle $i+1$ by intervehicular communication, and trigger the feedforward control of the following vehicle $i+1$.

If the boundedness is appropriately chosen to distinguish the initial tracking errors (large amplitude, low frequency) and unmodeled dynamics (small amplitude, high frequency), the feedforward control and intervehicular communication will be only triggered necessarily and infrequently.

IV. Analysis of Control Performances

A. Analysis of Control Objectives

Three theorems are proposed for the achievement of the three control objectives.

Theorem 1 (Individual Stability): The nominal system (7) is individual stable supposing that Assumption 4 holds.

Theorem 2 ($\mathcal{L}_p$ String Stability): The nominal system (7) is $\mathcal{L}_p$ string stable supposing that Assumption 4 holds.

Theorem 3 (DSS): The actual system (9) is disturbance $\mathcal{L}_p$ string stable supposing that Assumption 4 holds.

Proof: See Appendix A. □

B. Analysis of Control Efficiency

The theorem of the convexity of Problem 2 is proposed.

Theorem 4: The problem 2 is a convex optimization problem.

Proof: See Appendix B. □

By Theorem 4, Problem 2 can be solved computational efficiently by convex optimization tools. According to the integrated mechanism, the feedforward control and intervehicular communication are infrequently triggered by large amplitude disturbances. Therefore, the computation cost and communication cost of the proposed method are much smaller than MPC-like methods, which trigger the feedforward control and intervehicular communication at each control interval.

V. Simulations

In this section, we provide simulation results of the proposed method for a vehicular platoon equipped with CACC. The environmental settings are described in Table II. Two scenarios are designed as shown in Table III. The small amplitude disturbances (i.e., unmodeled dynamics) are generated by a truncated normal distribution for both scenarios. One large amplitude disturbance (i.e., initial tracking error) occurs only at time 0 s in the scenario 1, while three large amplitude disturbances occur at time 0, 2.5, 5 s in the scenario 2.

TABLE II
Environmental Settings

Parameters	Values	Parameters	Values
τ, τ_i	0.5	d_{min}	3
N_p	40	a_{max}	5
v_{max}	35	h_i	1

TABLE III
Simulation Scenarios

Parameters	Scenario 1	Scenario 2
Initial Tracking Error	10 m at time 0 s	10, 3, 3 m at time 0, 2.5, 5 s
Unmodeled Dynamics	A truncated normal distribution with $\omega_i \leq 0.1$	
Time Delay	The time delay is less than τ	

Fig. 3 shows the validation of the control objectives. Fig. 3(a) shows the mRPI determined by (19). As shown in Fig. 3(b) and (d), the platoon is disturbance $\mathcal{L}_p$ string stable. Fig. 3(c) illustrates the trajectories of (e_{s1}, e_{v2}), in which the dashed line denotes the nominal trajectory and the solid line denotes the actual trajectories. The trend of the actual system is dominated by nominal trajectory (i.e., the feedforward control) when range error is large and dominated by the mRPI (i.e., the feedback control) when the error is small.

Figs. 4 and 5 show the validation of the control efficiency. An MPC method is implemented as the benchmark. The MPC method solves Problem 2 with $\mathbb{Z} = \varnothing$ at each control interval. To obtain the required information, the intervehicular communication of the platoon is triggered at each control interval. Both the MPC method and the proposed method are implemented at MATLAB 2018a with Intel i5-6200U and 16G ARM. As shown in Fig. 4, both the MPC and the proposed method can achieve the control objectives similarly. In both the scenarios, the range errors of the platoon can converge to the equilibrium state satisfying the disturbance string stability requirements [(15)]. Considering the inputs of the methods, the proposed method shows a kind of conservativeness, where the input of MPC can reach the maximal input ($u = 5$), while the maximal input of the proposed method is a little smaller ($u = 4.6$). The conservativeness comes from the mRPI set, which is designed for the boundedness of new disturbances $\mathbb{W}$. If the new disturbance is smaller than that boundedness, the conservativeness will exist. It is the reasonable cost considering the much-improved performances of efficiency. As shown in Fig. 5, the proposed method is much more efficient than the MPC method regarding computation time and intervehicular communication times. Fig. 5(c) shows the tendency of communication number of times with the average time interval between two consecutive large amplitude disturbances (i.e., 3 m).

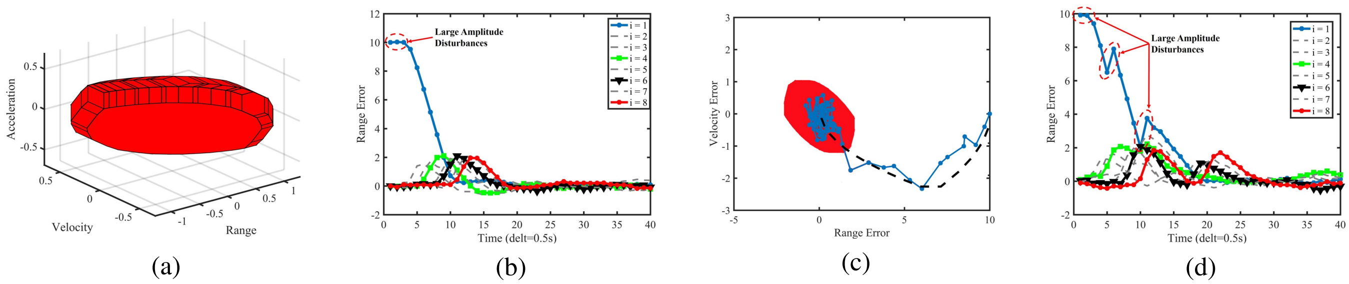

Fig. 3. Validation of the control objectives for the proposed method. (a) mRPI. (b) Scenario 1. (c) Illustration of (e_{s1}, e_{v2}) trajectories. (d) Scenario 2.

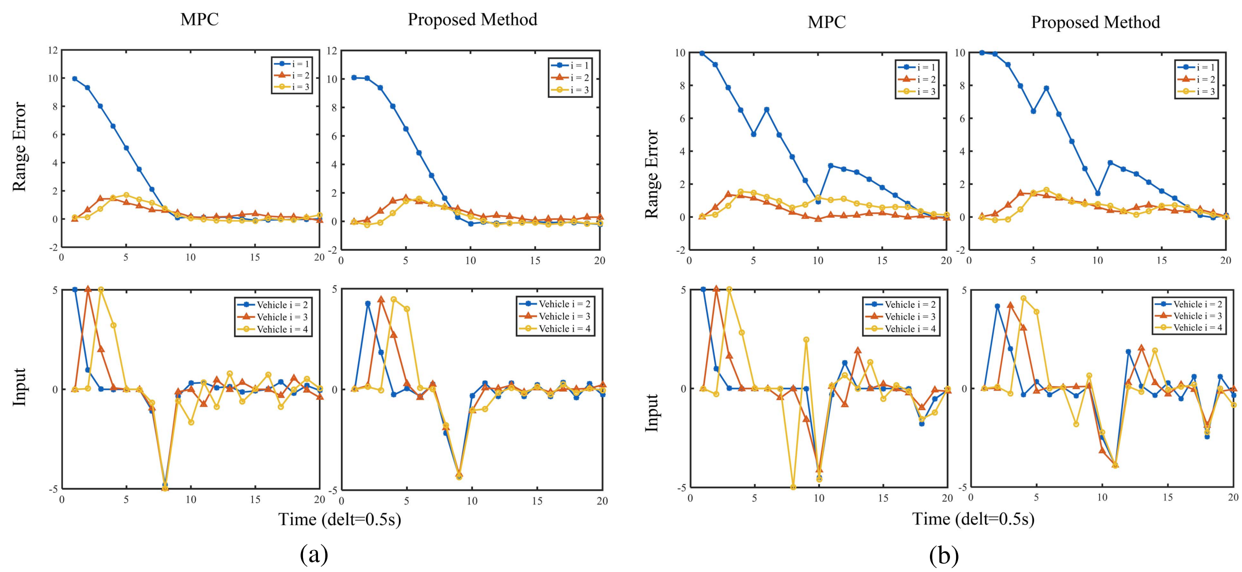

Fig. 4. Comparisons of the range error and input between the MPC method and the proposed method for (a) scenario 1 (b) and scenario 2.

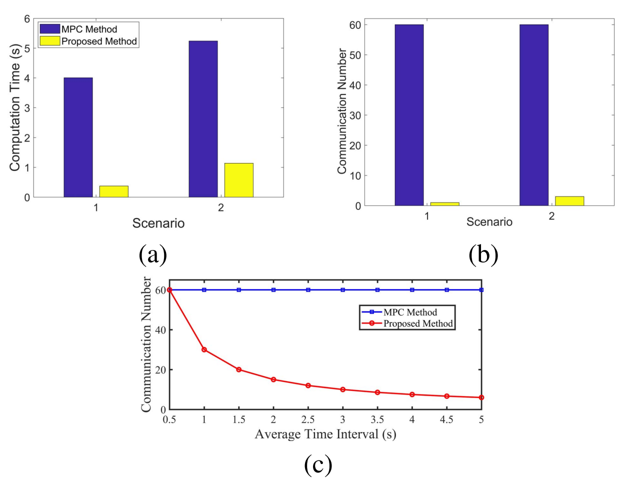

Fig. 5. Comparisons between (a) MPC method and the proposed method for computation time, (b) communication number of times with scenarios, and (c) communication number of times with the average time interval between two consecutive large amplitude disturbances.

VI. Conclusion

In this brief, a tube-based discrete control method is proposed to guarantee DSS-satisfying saturation constraints. Compared with MPC-like methods, the proposed method is efficient regarding computation and intervehicular communication.

The integration of the feedforward control and the feedback control has the advantage of utilizing planned information. In addition to large amplitude disturbances, other planned information, e.g., speed guidance from the intersection control, can also be utilized by this method. The existing methods of the feedback control could be improved by the integration of the feedforward control.

In the future work, the integration mechanism has the potential for further problems, e.g., general communication typology, communication equality [23], mixed traffic, and stochastic disturbances [34], [35].

Appendix A
Proof of Theorem 1–3

Lemma 1 [32]: If A_k is asymptotically stable, there exists RPI set of system (17) with bounded disturbances.

Lemma 2 [24]: Suppose that Assumption 4 holds. The nominal system is controlled with the requirements of the tight sets as $\overline{x}_i(k) \in \overline{\mathbb{X}}_i, \overline{u}_i(k) \in \overline{\mathbb{U}}_i,$, and the feedback controller is as shown in (17), then the actual system (9) satisfies $x_i(k) \in \mathbb{X}_i, u_i(k) \in \mathbb{U}_i, i \in S_m$, for $k = 0, \ldots, N_p$.

Proof of Theorem 1: The terminal sets $\overline{\mathbb{X}}_{i,f}, \overline{\mathbb{U}}_{i,f}$ for the nominal system are satisfied by the optimization problem 2. If there is no new large amplitude disturbance, the vehicle i will reach the terminal sets $\overline{\mathbb{X}}_{i,f}, \overline{\mathbb{U}}_{i,f}$ before the terminal time. Then, the vehicle i will stay at the equilibrium point, i.e., the system is individual stable. □

Proof of Theorem 2: The string stability constraints for the nominal system are satisfied by the optimization problem 2. The vehicle i will satisfy the $\mathcal{L}_p$ string stability requirements according to the Definition 2, i.e., the nominal system is $\mathcal{L}_p$ string stable. □

Proof of Theorem 3: By Lemma 1 and Lemma 2, for bounded disturbances $\omega_i(k)$, there is an RPI set $\mathbb{Z}$ and a constant $z > 0$, such that $\|\tilde{e}_{si}\|_{\mathcal{L}_\infty} \leq z$, for all $i \in \mathbb{S}_{e,m}$. Then, for $p \geq 1$, we obtain that

$$\begin{aligned}\|e_{si}\|_{\mathcal{L}_p} &= \|\overline{e}_{si} + \tilde{e}_{si}\|_{\mathcal{L}_p} \\ &\leq \|\overline{e}_{si}\|_{\mathcal{L}_p} + \|\tilde{e}_{si}\|_{\mathcal{L}_p} \\ &\leq \alpha(|e_{s1}(0)|) + z \\ &= \alpha(|e_{s1}(0)|) + k_w c_w \end{aligned} \tag{28}$$

where $k_w = z/c_w$ is independent of platoon size. The first inequality is obtained by the triangle inequality of the norm and the second inequality is obtained by the Theorem 2. According to the Definition 3, the actual system is DSS. □

Appendix B
Proof of Theorem 4

Lemma 3 [26]: If W is a polytope, the mRPI set estimated by (19) is convex.

Proof of Theorem 4: First, the objective function J_i [(25)], constraints sets $\mathbb{X}_i, \mathbb{U}_i$ [(11)], mRPI set $\mathbb{Z}$, tight sets $\overline{\mathbb{X}}_i, \overline{\mathbb{U}}_i$, terminal constraints sets $\overline{\mathbb{X}}_{i,f}, \overline{\mathbb{U}}_{i,f}$, and the state-space equations of nominal system [(7)] are convex sets.

Therefore, the theorem is proven if the sets of string stability requirements, i.e., $\overline{\mathbb{S}}_i$, are convex. As defined in (24), $\overline{\mathbb{S}}_i$ are convex if and only if $g(\overline{u}_i(k), p) = \|\overline{e}_{si}\|_{\mathcal{L}_p}$ is a convex function. By the definition of $\mathcal{L}_p$-norm, we have

$$g(\overline{u}_i(k), p) = \left(\int_0^{N_p\cdot\tau} |\overline{e}_{si}(\overline{u}_i(k), t)|^p \, dt\right)^{1/p} \tag{29}$$

where

$$\overline{e}_{si}(\overline{u}_i(k), t) = \overline{e}_{si}(k) + \overline{e}_{vi}(k)\cdot(t - k\tau) + (\overline{u}^a_{i-1}(k) - \overline{u}_i(k)) \cdot (t - k\tau)^2$$

for $k\tau \leq t < (k+1)\tau$. The first equivalence is obtained by the terminal constraints. Note that $\overline{e}_{si}(k), \overline{e}_{vi}(k)$ are linear combination of $\overline{u}_i(0), \ldots, \overline{u}_i(k-1)$ [see (7)]. Therefore, $\overline{e}_{si}(\overline{u}_i(k), t)$ is a convex function in decision variables $\overline{u}_i(0), \ldots, \overline{u}_i(k)$, $k = 0, \ldots, N_p$, for any $t \in [0, N_p\tau]$. Because the $\mathcal{L}_p$-norm of a convex function is still a convex function [36], [37], we obtain that $g(\overline{u}_i(k), p)$ is a convex function in decision variables $\overline{u}_i(0), \ldots, \overline{u}_i(k)$, for any $t \in [0, 0 + N_p\tau]$. By the constraints of (24), the convexity of the sets $\overline{\mathbb{S}}_i$ is followed, which concludes the theorem. □

References

[1] B. van Arem, C. J. G. van Driel, and R. Visser, "The impact of cooperative adaptive cruise control on traffic-flow characteristics," *IEEE Trans. Intell. Transp. Syst.*, vol. 7, no. 4, pp. 429–436, Dec. 2006.

[2] L. Li, D. Wen, and D. Yao, "A survey of traffic control with vehicular communications," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 425–432, Feb. 2014.

[3] G. J. L. Naus, R. P. A. Vugts, J. Ploeg, M. J. G. van de Molengraft, and M. Steinbuch, "String-stable CACC design and experimental validation: A frequency-domain approach," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4268–4279, Nov. 2010.

[4] S. Öncü, J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Cooperative adaptive cruise control: Network-aware analysis of string stability," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 4, pp. 1527–1537, Aug. 2014.

[5] D. Swaroop and J. K. Hedrick, "String stability of interconnected systems," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 349–357, Mar. 1996.

[6] S. Sheikholeslam and C. A. Desoer, "Longitudinal control of a platoon of vehicles," in *Proc. Amer. Control Conf.*, May 1990, pp. 291–296.

[7] J. Ploeg, D. P. Shukla, N. van de Wouw, and H. Nijmeijer, "Controller synthesis for string stability of vehicle platoons," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 854–865, Apr. 2014.

[8] J. Ploeg, N. van de Wouw, and H. Nijmeijer, "Lp string stability of cascaded systems: Application to vehicle platooning," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 2, pp. 786–793, Mar. 2014.

[9] J. I. Ge and G. Orosz, "Dynamics of connected vehicle systems with delayed acceleration feedback," *Transp. Res. C, Emerg. Technol.*, vol. 46, pp. 46–64, Sep. 2014.

[10] G. Orosz, "Connected cruise control: Modelling, delay effects, and nonlinear behaviour," *Vehicle Syst. Dyn.*, vol. 54, no. 8, pp. 1147–1176, 2016.

[11] Z. Li, Z. Duan, and G. Chen, "On H_∞ and H_2 performance regions of multi-agent systems," *Automatica*, vol. 47, no. 4, pp. 797–803, 2011.

[12] F. Gao, S. E. Li, Y. Zheng, and D. Kum, "Robust control of heterogeneous vehicular platoon with uncertain dynamics and communication delay," *IET Intell. Transport Syst.*, vol. 10, no. 7, pp. 503–513, Sep. 2016.

[13] L. Zhang and G. Orosz, "Motif-based design for connected vehicle systems in presence of heterogeneous connectivity structures and time delays," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1638–1651, Jun. 2016.

[14] B. Besselink and K. H. Johansson, "String stability and a delay-based spacing policy for vehicle platoons subject to disturbances," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4376–4391, Sep. 2017.

[15] V. S. Dolk, J. Ploeg, and W. P. M. H. Heemels, "Event-triggered control for string-stable vehicle platooning," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 12, pp. 3486–3500, Dec. 2017.

[16] Y. Zheng, S. E. Li, J. Wang, D. Cao, and K. Li, "Stability and scalability of homogeneous vehicular platoon: Study on the influence of information flow topologies," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 1, pp. 14–26, Jan. 2016.

[17] W. B. Dunbar and D. S. Caveney, "Distributed receding horizon control of vehicle platoons: Stability and string stability," *IEEE Trans. Autom. Control*, vol. 57, no. 3, pp. 620–633, Mar. 2012.

[18] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Eng. Pract.*, vol. 11, no. 7, pp. 733–764, 2003.

[19] F. Li and Y. Wang, "Cooperative adaptive cruise control for string stable mixed traffic: Benchmark and human-centered design," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 12, pp. 3473–3485, Dec. 2017.

[20] Y. Zheng, S. E. Li, K. Li, F. Borrelli, and J. K. Hedrick, "Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 3, pp. 899–910, May 2017.

[21] D. Moser, R. Schmied, H. Waschl, and L. del Re, "Flexible spacing adaptive cruise control using stochastic model predictive control," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 1, pp. 114–127, Jan. 2018.

[22] D. Mayne, "Robust and stochastic model predictive control: Are we going in the right direction?" *Annu. Rev. Control*, vol. 41, pp. 184–192, 2016.

[23] G. Guo and S. Wen, "Communication scheduling and control of a platoon of vehicles in VANETs," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1551–1563, Jun. 2016.

[24] D. Q. Mayne and W. Langson, "Robustifying model predictive control of constrained linear systems," *Electron. Lett.*, vol. 37, no. 23, pp. 1422–1423, Nov. 2001.

[25] W. Langson, I. Chryssochoos, S. Raković, and D. Q. Mayne, "Robust model predictive control using tubes," *Automatica*, vol. 40, no. 1, pp. 125–133, 2004.

[26] S. V. Rakovic, E. C. Kerrigan, K. I. Kouramas, and D. Q. Mayne, "Invariant approximations of the minimal robust positively Invariant set," *IEEE Trans. Autom. Control*, vol. 50, no. 3, pp. 406–410, Mar. 2005.

[27] S. Sheikholeslam and C. A. Desoer, "Longitudinal control of a platoon of vehicles with no communication of lead vehicle information: A system level study," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 546–554, Nov. 1993.

[28] S. S. Stankovic, M. J. Stanojevic, and D. D. Siljak, "Decentralized overlapping control of a platoon of vehicles," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 5, pp. 816–832, Sep. 2000.

[29] A. Ghasemi, R. Kazemi, and S. Azadi, "Stable decentralized control of a platoon of vehicles with heterogeneous information feedback," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4299–4308, Nov. 2013.

[30] K. Ogata, *Discrete-time Control Systems*, vol. 8. Englewood Cliffs, NJ, USA: Prentice-Hall, 1995.

[31] D. J. Bender and A. J. Laub, "The linear-quadratic optimal regulator for descriptor systems: Discrete-time case," *Automatica*, vol. 23, no. 1, pp. 71–85, 1987.

[32] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, Nov. 1999.

[33] J. M. Maestre and R. R. Negenborn, *Distributed Model Predictive Control Made Easy*, vol. 69. Dordrecht, The Netherlands: Springer, 2013.

[34] G. Guo and W. Yue, "Autonomous platoon control allowing range-limited sensors," *IEEE Trans. Veh. Technol.*, vol. 61, no. 7, pp. 2901–2912, Sep. 2012.

[35] G. Guo and W. Yue, "Sampled-data cooperative adaptive cruise control of vehicles with sensor failures," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 6, pp. 2404–2418, Dec. 2014.

[36] R. A. Adams and J. J. Fournier, *Sobolev Spaces*, vol. 140. New York, NY, USA: Academic, 2003.

[37] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.