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String stability for vehicular platoon control: Definitions and analysis methods

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ABSTRACT

The platooning of connected and automated vehicles (CAVs) is expected to have a transformative impact on road transportation, e.g, enhancing highway safety, improving traffic efficiency, and reducing fuel consumption. One critical task of platoon control is to achieve string stability, for which various models and methods had been proposed. However, different types of definitions and analysis methods for string stability were proposed over the years and were not thoroughly compared. To fill these gaps, this paper aims to clarify the relationship of ambiguous definitions and various analysis methods, providing a rigorous foundation for future studies. A series of equivalences are summarized and discussed. The pros and cons of different analysis methods and definitions are discussed, too. All these discussions provide insights for practical selection of analyzing methods for vehicle platoons.

Contents

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1. Introduction

Vehicular platoon control, as an effective way of improving traffic efficiency, has attracted extensive interest (Guanetti, Kim, & Borrelli, 2018; Horowitz & Varaiya, 2000; Ioannou & Chien, 1993; Li, Zheng, Li, & Wang, 2015; Sheikholeslam & Desoer, 1990; Shladover, 1995). A system of vehicular platoon is a string of two or more closely vehicles traveling with desired cruising velocity and distance. Compared with human drivers, an automatic platoon control system has the advantages of decreasing intervehicle distance (i.e., tight formation), and is thus considered a promising solution for reducing traffic congestion, aerodynamic drag, and fuel consumption (Al Alam, Gattami, & Johansson, 2010; Chien & Ioannou, 1992; Li & Chen, 2017).

The tight formation control of platoons has a particular difficulty known as "string instability", i.e., disturbances of system states are amplified along the string of vehicles, as shown in Fig. 1(b) (from Peppard (1974)). As demonstrated by observations (Treiterer & Myers, 1974) and experiments (Sugiyama et al., 2008), the string instability of tight formation platoons can cause the emergence of a jam (e.g., stop and go) without bottleneck in both circuits and highways, which seriously compromises the benefits of platoon control.

To resolve this problem, the property of string stability has been widely studied for platoon control. Intuitively, a platoon is said to be string stable if it has the property, i.e., the disturbances are not amplified when propagating along the vehicle string (Peppard, 1974), as shown in Fig. 1(c). The basic process of string stability studies can be divided into three steps: (1) mathematically define the property of string stability; (2) derive sufficient conditions by analysis methods; (3) design controllers to satisfy the sufficient conditions.

1.1. Motivation

A vast amount of literature has proposed various types of definitions and analysis methods. In many studies, although the simulation results show similar property to that in Fig. 1(c), the defined properties of string stability have confusing discrepancies. Various definitions have been described by different domains (e.g., frequency-domain and time-domain), norms (e.g., $\mathcal{L}_2$, $\mathcal{L}_p$, and $\mathcal{L}_\infty$), strength (e.g., weak and strict). The ambiguous definitions block the comparison among different studies. The rigorous analysis of their relations is required for further study. Moreover, a lot of analysis methods have been proposed and derived many alternative properties. Their relations, pros and cons, and solvable problems are not yet discussed much. The better understanding of these methods and properties is the foundation to further studies of troublesome issues.

1.2. Scope and aim

This paper focuses on the definitions of string stability and the analysis methods which derive alternative properties of string stability. To better explain the related conceptions, we will briefly introduce the studies of vehicular platoon control. To make the research concise, however, we will not discuss other problems in the field of vehicular platoon control (Li et al., 2015; Li, Zheng et al., 2017) in this paper.

The aims of this paper are: (1) to clarify the relations of ambiguous definitions of string stability and recommend a unified definition; (2) to discuss the relations, pros and cons, and solvable problems of various analysis methods and recommend methods for existing troublesome issues; and (3) to dig into the relations of the derived alternative properties, which provide insights of solutions to combat string instability.

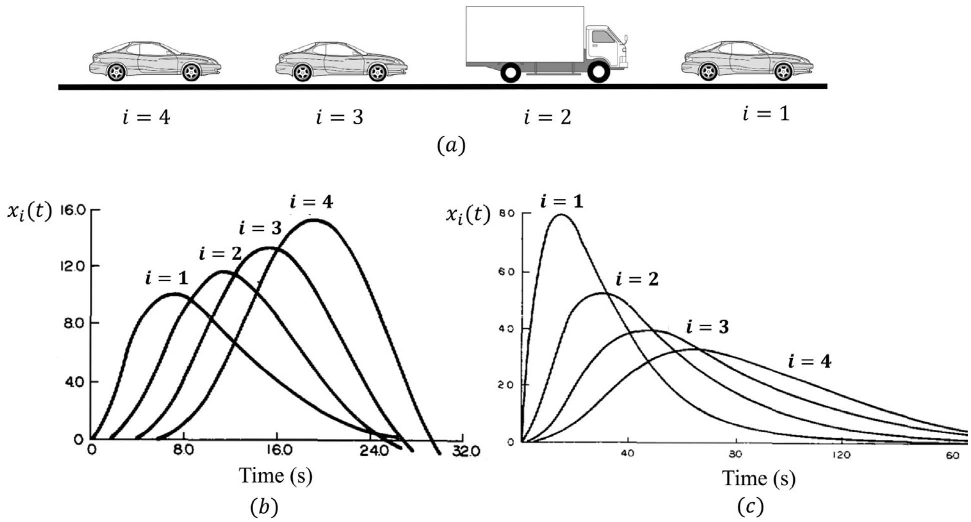

Fig. 1. Illustration of a platoon system (a) and intuitive descriptions of string stability (b-c) (Peppard, 1974), where $x_i(t)$ denotes the state fluctuation (e.g., position error) of vehicle i at time t.

1.3. Contributions

The major contributions of this paper are as follows:

First, the relations of ambiguous definitions are rigorously analyzed. The commonly used definitions are introduced and compared. Three essential properties are summarized for string stability, i.e., convergence, boundedness, and scalability. Analogous to stability definitions in control theory, three types of string stability definitions, i.e., Lyapunov, input-to-output, and input-to-state string stability, are proposed as the bridges of the relations. A rigorous analysis of these string stability definitions is elaborated in Theorem 1. Inspired by the theorem, the proposed definition, i.e., input-to-state string stability (ISSS), is recommended for future studies. The justifications of ISSS are also provided. This paper extends and deepens the discussions on definitions of the previous major survey in this field (Ploeg, Van De Wouw, & Nijmeijer, 2014; Stüdli, Seron, & Middleton, 2017).

Second, the various analysis methods are compared, and their derived alternative properties are rigorously analyzed. The methods are classified into three families, i.e., time-, z-, and s- domain analysis methods. The analysis tasks are divided into the temporal and spatial perspectives. The cons and pros of these methods are discussed by these perspectives, based on which we recommend methods for existing troublesome issues. Moreover, the derived alternative properties are rigorously analyzed in Theorem 2, which shows the relations among these properties and the recommended definition (i.e., ISSS) for the commonly studied platoon system. The common solutions to combat string instability are compared with the "weak coupling property".

1.4. Structure

The remainder of this paper is arranged as follows. Section 2 describes the preliminaries of the study. Section 3 briefly introduces the vehicular platoon control that led to the studies of string stability. The commonly used definitions of string stability are introduced in Section 4 and their relations are analyzed in Section 5. Section 6 introduces the common analysis methods and their relations. The alternative properties derived by these methods are discussed in Section 7. The future directions are discussed in Sections 8 and 9 concludes the paper.

2. Preliminaries

The field of a real number is denoted by $\mathbb{R}$, whereas $\mathbb{N}=\{1,2,\dots\}$. For a vector $x\in\mathbb{R}^n$, its p-norm is given as

$$\|x\|_p=\left(\sum_{i=1}^{n}|x_i|^p\right)^{1/p},\ p\in[1,\infty) \tag{1}$$

$$\|x\|_\infty=\max_i |x_i|. \tag{2}$$

Given a Lebesgue measurable signal $x(t): I\to\mathbb{R}^n$, $\|x\|^I_{\mathcal{L}_p}$ denotes its $\mathcal{L}_p$ norm defined as

$$\|x\|^I_{\mathcal{L}_p}=\left(\int_I \|x(t)\|_p^p dt\right)^{1/p}<\infty,\ p\in[1,\infty) \tag{3}$$

$$\|x\|^I_{\mathcal{L}_\infty}=\sup_{t\in I}\|x\|_\infty, \tag{4}$$

where the shorthand notation $\|x\|_{\mathcal{L}_\infty}=\|x\|^{[0,\infty)}_{\mathcal{L}_\infty}$ is used when $I=[0,\infty)$. Given a transfer function $G(j\omega)$ of the system, the $\mathcal{H}_\infty$ norm of the system is defined as

$$\|G\|_{\mathcal{H}_\infty}=\sup_\omega |G(j\omega)|,\ \text{(SISO)} \tag{5}$$

$$\|G\|_{\mathcal{H}_\infty}=\sup_\omega \bar{\sigma}(G(j\omega)),\ \text{(MIMO)} \tag{6}$$

where SISO denotes a single-input and single-output system, while MIMO denotes a multiple-input and multiple-output system. $\bar{\sigma}(G(j\omega))$ is the maximum singular value of the matrix $G(j\omega)$ (see Zhou, Doyle, Glover et al. (1996) for details). A continuous function $\alpha:[0,a)\to[0,\infty), a\in\mathbb{R}^+$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0)=0$. A continuous function β: $[0, a)\times[0,\infty)\to[0,\infty)$ is said to be of class $\mathcal{KL}$ if, for each fixed s, the function $\beta(\cdot, s)$ is of class $\mathcal{K}$, and for each fixed r, $\beta(r, \cdot)$ is decreasing and satisfies $\beta(r, s)\to 0$ as $s\to\infty$. We say $x\in\mathcal{L}_\infty$ if $\|x\|_{\mathcal{L}_\infty}<\infty$.

In general, we consider a platoon system

$$\begin{aligned}\dot{x}&=f(x,\omega),\\ y&=g(x),\end{aligned} \tag{7}$$

where the function $f(x,\omega):\mathbb{R}^{mn}\times\mathbb{R}^m\to\mathbb{R}^{mn}$ is continuously differentiable and globally Lipschitz in (x, ω), $g(x):\mathbb{R}^{mn}\to\mathbb{R}^m$ is continuously differentiable and global Lipschitz in x. $x\in\mathbb{R}^{mn}$ denotes

Table 1
Notations of the used variables.

Variables	Notations
S_m	$S_m = \{0\} \cup \{i \in \mathbb{N} \| 1 \le i \le m-1\}$.
$S_{e,m}$	$S_{e,m} = \{i \in \mathbb{N} \| 1 \le i \le m-1\}$.
$x_i(t)$	$x_i(t) \in \mathbb{R}^n$ denotes the state of ith subsystem.
$\omega_i(t)$	$\omega_i(t) \in \mathbb{R}$ denotes the disturbance of the ith subsystem.
$y_i(t)$	$y_i(t) \in \mathbb{R}$ denotes the output of the ith subsystem.
$x(t)$	$x(t) \in \mathbb{R}^{mn}$ denotes the system state.
$\omega(t)$	$\omega(t) \in \mathbb{R}^m$ denotes the system disturbances.
$y(t)$	$y(t) \in \mathbb{R}^m$ denotes the system outputs.
$X_i(s)$, $Y_i(s)$, $U_i(s)$	The Laplace transforms of $x_i(t)$, $y_i(t)$, and $u_i(t)$.
$X(s)$, $Y(s)$, $U(s)$	The Laplace transforms of $x(t)$, $y(t)$, and $u(t)$.
$\mathcal{X}(z), \mathcal{Y}(z), \mathcal{U}(z)$	The bilateral z-transforms of $x(t)$, $y(t)$, and $u(t)$.

the system state vector, and $\omega \in \mathbb{R}^m$ denotes the disturbances. We assume the origin $x = 0$ is an stable equilibrium point for the unforced system $\dot{x} = f(x, 0)$. For platoon systems, $m \in \mathbb{N}$ denotes the platoon length, and $n \in \mathbb{N}$ denotes the state orders of subsystems. The notations are listed in Table 1.

3. Vehicular platoon control

This section briefly introduces the control problem for vehicular platoon that led to the studies of string stability. The platoon control problem, originally proposed by Levine and Athans (1966), studies how to design a controller to achieve control objectives of a platoon system. This includes platoon system description, control objectives, and controller design methods. The background of the two focuses of this paper, i.e., definition and analysis methods of string stability, is introduced.

3.1. Platoon system description

The description of a platoon system specifies the "scenario" of string stability studies. The four-component framework (Li et al., 2015) is adopted in this paper, i.e., node dynamics, information flow topology, distributed controller, and formation geometry. Furthermore, communication quality and disturbances of the system are emphasized. A specific platoon system can be identified by these six components. The abbreviations in this subsection are listed in Table A.1.

3.1.1. Node dynamics (ND)

The ND component denotes the vehicle longitudinal dynamics. According to modeling formation, the vehicle dynamics can be classified into nonlinear (Dunbar & Caveney, 2012; Rajamani, 2011), second-order model (Naus, Vugts, Ploeg, van de Molengraft, & Steinbuch, 2010; Yanakiev & Kanellakopoulos, 1996), third-order model (Godbole & Lygeros, 1994; Liang & Peng, 1999; Warnick & Rodriguez, 2000), and general linear model Liang and Peng (2000); Seiler, Pant, and Hedrick (2004).

3.1.2. Information flow topology (IFT)

The IFT component describes how vehicles exchange information with others. Fig. 2 shows commonly used topologies, including predecessor following (PF) (Naus et al., 2010), predecessor leader following (PLF) (Sheikholeslam & Desoer, 1990; Swaroop & Hedrick, 1999), bidirectional (BD) (Eyre, Yanakiev, & Kanellakopoulos, 1997; Ghasemi, Kazemi, & Azadi, 2013; Knorn, Donaire, Agüero, & Middleton, 2014; Yanakiev & Kanellakopoulos, 1996), bidirectional-leader (BDL) (Zheng, Li, Wang, Cao, & Li, 2016), two-predecessor following (TPF) (Swaroop & Hedrick, 1999), and two-predecessor-leader following (TPLF) (Li et al., 2015). More general IFTs can also be applied, e.g., r-predecessor following (rPF) and r-predecessor-leader-following (rPLF), where r denotes the number of predecessors with communication.

3.1.3. Distributed controller (DC)

The DC component describes the controller of the platoon system for achieving control objectives, e.g., linear controller (Naus et al., 2010; Sheikholeslam & Desoer, 1993), optimal controller (Chu, 1974b; Jin & Orosz, 2017; Liang & Peng, 1999), $\mathcal{H}_\infty$ controller (Ploeg, Shukla, van de Wouw, & Nijmeijer, 2014), Model Predictive Control (MPC) (Dolk, Ploeg, & Heemels, 2017; Dunbar & Caveney, 2012), and Sliding-mode Control (SMC) (Fernandes & Nunes, 2012).

3.1.4. Formation geometry (FG)

The FG component denotes the desired inter-vehicle distance of the platoon system, also known as range policy in many studies. Three major policies exist, i.e., constant distance (CD) policy (Liu, Goldsmith, Mahal, & Hedrick, 2001; Sheikholeslam & Desoer, 1993), constant time headway (CTH) policy (Chien & Ioannou, 1992; Zhou & Peng, 2005), and nonlinear distance (NLD) policy (Orosz, 2016; Santhanakrishnan & Rajamani, 2003).

3.1.5. Communication quality (CQ)

The CQ component describes the issues caused by communication quality, i.e., communication time delay (di Bernardo, Salvi, & Santini, 2015; Liu et al., 2001; Oncu, Van de Wouw, Heemels, & Nijmeijer, 2012; Qin, Gomez, & Orosz, 2017; Xiao, Darbha, & Gao, 2008; Xiao, Gao, & Wang, 2009) and packet loss (Moreau, 2005; Ploeg, Semsar-Kazerooni, Lijster, van de Wouw, & Nijmeijer, 2015; Teo, Stipanovic, & Tomlin, 2002; 2003).

3.1.6. Disturbances

The disturbances denote causes of the deviation of platoon systems. The commonly used types of disturbances include:

- Type I: Initial condition perturbations for the leading vehicle;
- Type II: Initial condition perturbations for all vehicles;
- Type III: External disturbances for leading vehicle with zero initial condition perturbations;
- Type IV: External disturbances for all vehicles with zero initial condition perturbations.

Finally, two important terminologies of platoon systems are introduced, i.e., homogeneity and infinity. The platoon is said to be "homogeneous" if all vehicles have identical dynamics (Chu, 1974a); otherwise it is called "heterogeneous" (Naus et al., 2010). Moreover, the platoon is said to be "infinite" if the platoon has infinite number of vehicles (Swaroop & Hedrick, 1996); otherwise it is called "finite" (Jin & Orosz, 2014). Benefiting from the tractability of theoretical analysis, the infinite platoons are commonly studied to capture the essence of the large-but-finite platoons (Curtain, Iftime, & Zwart, 2010; Jovanovic & Bamieh, 2005) and scalability of the platoon.

3.2. Control objectives

The control of autonomous platoon aims to ensure all the vehicles in the same group to move at a consensual speed while maintaining the desired spaces between adjacent vehicles, and thus increase traffic capacity, improve traffic safety, and reduce fuel consumption (Horowitz & Varaiya, 2000). Stability properties of the platoon system are the foundation of all above-mentioned control objectives. Two types of stability have been proposed, i.e., individual stability and string stability.

Individual stability describes vehicles converging to given trajectories (Dolk et al., 2017; Dunbar & Caveney, 2012; Ghasemi et al., 2013; Guo, Ding, & Han, 2014; Ploeg,Shukla et al., 2014; Swaroop & Hedrick, 1999; Zheng, Li, Wang et al., 2016) and convergence speed is characterized by a stability margin (Barooah, Mehta, & Hespanha, 2009; Hao & Barooah, 2012; Hao, Barooah, & Mehta, 2011; Zheng, Li, Li, & Wang, 2016). However, an individual stable

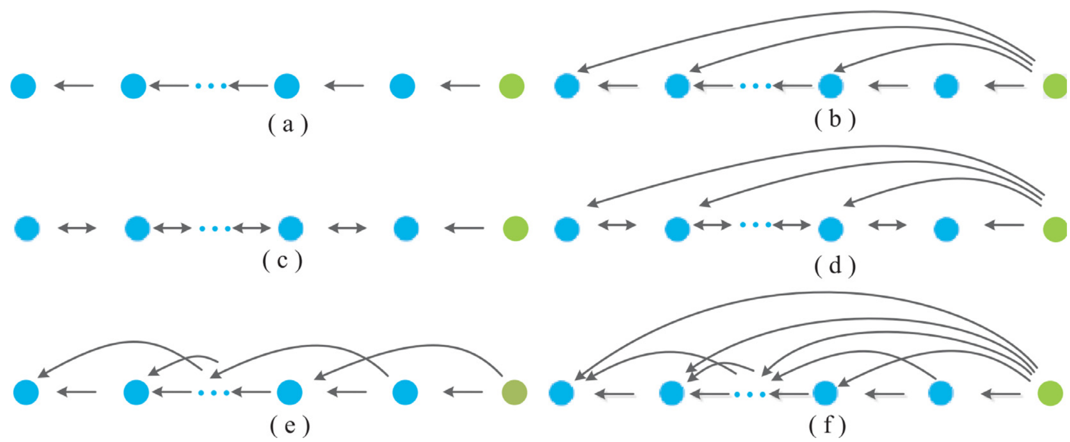

Fig. 2. Typical IFTs for platoon, where the green circle denotes the leading vehicle. (a) PF; (b) PLF; (c) BD; (d) BDL; (e) TPF; (f) TPLF (Li et al., 2015). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

platoon can amplify a small disturbance and cause a traffic jam (e.g., stop-and-go) without bottlenecks as the increase of the platoon length (Hedrick, Tomizuka, & Varaiya, 1994; Naus et al., 2010; Shaw & Hedrick, 2007b; Sugiyama et al., 2008).

To solve this issue, string stability was proposed and extensively studied. Intuitively, a platoon is said to be string stable if the disturbances are not amplified when propagating downstream along the vehicle string (Peppard, 1974). To mathematically define this property, various formal definitions of string stability have been proposed for different platoon systems in the past few decades, e.g., (Jin & Orosz, 2014; Khatir & Davidson, 2004; Li, Shi, & Yan, 2016; Ploeg,Van De Wouw et al., 2014; Swaroop & Hedrick, 1996). The details of these definitions will be discussed in the next section, and their relations are one focus of this paper.

3.3. Controller design methods

The key to designing a platoon controller is to achieve the control objectives, e.g., individual stability and string stability.

One most commonly used family design methods is to predetermine a feedback controller and tune its parameters to achieve the control objectives. The string stability property is redefined mathematically as certain conditions of transfer functions (Jin & Orosz, 2014; Naus et al., 2010; Orosz, 2016) or Lyapunov functions (Besselink & Johansson, 2017; Dolk et al., 2017; Swaroop & Hedrick, 1996). The common limitation of these methods is the difficulty of explicitly satisfying saturation constraints, e.g., input saturation and safety constraints.

Another family of design methods is feedforwrd control (Li & Li, 2019; Liu, Li, Li, & Wang, 2017), e.g., decentralized Model Predictive Control (DMPC) methods (Dunbar & Caveney, 2012). The properties of individual stability and string stability are formulated as explicit constraints. By solving the formulated optimization problem at each control episode, the controller can guarantee the control objectives.

The critical step in both families of methods is to analyze the string stability property and derive its sufficient conditions. The system is necessarily string stable if its sufficient conditions are satisfied by the designed controller. Therefore, the analysis methods of string stability property are very critical, and are another focus of this paper.

4. A catalog of string stability definitions

This section introduces the commonly used definitions of string stability in the platoon control problem. The development of string stability definitions is closely related to the assumptions on platoon systems, and deeply influences the analysis methods. Therefore, the specific platoon system and analysis methods are briefly discussed when a new definition is introduced. The abbreviations of definitions are listed in Table A.2.

4.1. Original definition of string stability (OSS)

We first recall the original definition of string stability (OSS) given in Chu (1974a).

Definition 1. (OSS) A string of vehicle is stable if, for any set of bounded initial disturbances to all the vehicles, the position fluctuations of all the vehicles remain bounded and these fluctuations approach zeros as $t \to \infty$.

If type I or II disturbances exist and are bounded, Definition 1 (OSS) specifies two properties:

- boundedness of position fluctuations of all vehicles;
- convergence of position fluctuations of all vehicles.

To make the boundedness property non-trivial, it should hold if the platoon system has any number of vehicles, i.e.,

- the boundedness holds for any string length.

The invariance of the bounds under the string length is an important property, as it guarantees that the notion of string stability is scalable and allows for the addition or removal of vehicles from a string without affecting stability. In some cases, the convergence property is also called as individual stability.

OSS is very close to the intuitive description (Peppard, 1974) of string stability with the exception that the position fluctuations are not necessarily smaller than initial disturbances. The position fluctuations can be generalized to other system states fluctuations, e.g., velocity.

The major limitation of OSS is the theoretical intractability for generic platoon systems. The proposed z-transformation technique in Chu (1974a) can only be adopted to identical platoons with homogeneous ND, IFT, DC, FG, and CQ components.

4.2. SFSS and its variants FSS, ESS, HTS

For convenience of theoretical analysis, a necessary condition for string stability was proposed in frequency domain in Peppard (1974). This condition has been widely applied in the following string stability studies and is commonly considered as the definition of “strong” string stability (Naus et al., 2010). Therefore, we introduce this condition as a definition of strong frequency-domain string stability (SFSS).

Definition 2. (SFSS) For linear platoon systems with the PF topology, the system is SFSS if the transfer function of outputs between vehicle i and its predecessor $i-1$, denoted as $G_{i-1,i}$, is such that

$$\|G_{i-1,i}(j\omega)\|_{\mathcal{H}_\infty} \leq 1, \forall i \in S_{e,m}, \forall m \in \mathbb{N}. \tag{8}$$

One specific limitation of SFSS is the PF assumption on the platoon system. The SFSS is meaningless for other IFTs, e.g., BD, where Eq. (8) requires modification (Peppard, 1974). For rPF and rPLF topologies, three modified definitions were proposed, i.e., frequency-domain string stability (FSS) (Naus et al., 2010), eventual string stability (ESS) (Khatir & Davidson, 2004), and head-to-tail stability (HTS) (Jin & Orosz, 2014; 2017).

Definition 3. (FSS) For linear platoon system with rPF or rPLF topologies, the system is FSS if the transfer function of outputs between the leading vehicle 0 and any other vehicle i, denoted as $G_{0,i}$, is such that

$$\|G_{0,i}(j\omega)\|_{\mathcal{H}_\infty} \leq 1, \forall i \in S_{e,m}, \forall m \in \mathbb{N}. \tag{9}$$

Definition 4. (ESS) For linear platoon system with rPF or rPLF topology, the system is ESS if the transfer function of outputs between the leading vehicle 0 and any other vehicle i, denoted as $G_{0,i}$, and there exists $N < m$, is such that

$$\|G_{0,i}(j\omega)\|_{\mathcal{H}_\infty} \leq 1, \forall i > N, \forall m \in \mathbb{N}. \tag{10}$$

Definition 5. (HTS) For linear platoon system with rPF or rPLF topology, the system is HTS if the transfer function of outputs between the leading vehicle 0 and the tail vehicle m, denoted as $G_{0,m}$, is such that

$$\|G_{0,m}(j\omega)\|_{\mathcal{H}_\infty} \leq 1, \forall m \in \mathbb{N}. \tag{11}$$

FSS, ESS, and HTS assume the existence of a leading vehicle in platoons, which is reasonable for practical applications. The FSS is sometimes called as "weak" string stability, which is more flexible regarding IFTs compared with SFSS. The ESS is a special case of FSS, i.e., a system is ESS if it is FSS. The HTS was originally designed for mixed traffic, where the connected and automated vehicle was regarded as the tail vehicle (Jin & Orosz, 2014; 2017).

The first limitation of these definitions in frequency domain is the linear assumption on the platoon system. It does not mean the ND is assumed to be linear, but it can be made to appear linear through suitable nonlinear feedback techniques, e.g., Sheikholeslam and Desoer (1993), Stankovic, Stanojevic, and Siljak (2000), Ghasemi et al. (2013), Zheng, Li, Wang et al. (2016), which require a perfect prior knowledge of the vehicular dynamics.

The second limitation of these definitions is that only the type III disturbances are considered for platoon systems. This limitation comes from zero initial condition assumptions in the Laplace transform of these definitions. The system properties for other disturbance types require further analysis.

The third and more critical limitation is that $\left\|G_{i,j}(j\omega)\right\|_{\mathcal{H}_\infty} \leq 1$ only captures the signals in a perspective of "energy" (i.e., $\mathcal{L}_2$ norm), but not their maximal amplitudes (i.e., $\mathcal{L}_\infty$ norm). The details of their discrepancies will be discussed in the next section.

4.3. TSS and ATSS

To generalize the concept of string stability to a class of interconnected systems as

$$\dot{x}_i = f_i(x_i, x_{i-1}, \cdots, x_{i-r}), \tag{12}$$

where $i \in \mathbb{N}$, $x \in \mathbb{R}^n$, $x_{i-j} \equiv 0$ for $\forall$ $i \leq j$, and $f(0, \cdots, 0) = 0$, time-domain string stability (TSS) and asymptotically time-domain string stability (ATSS) were proposed in Swaroop and Hedrick (1996).

Definition 6. (TSS) The origin $x_i = 0, i \in \mathbb{N}$ of (12) is TSS, if given any $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\sup_i |x_i(0)| < \delta \Rightarrow \sup_i \|x_i(t)\|_\infty < \epsilon. \tag{13}$$

Definition 7. (ATSS) The origin $x_i = 0, i \in \mathbb{N}$ of (12) is ATSS, if it is TSS and $\sup_i \|x_i(t)\|_\infty \to 0$ asymptotically.

The interconnected systems in Eq. (12) specify the platoon system as rPF. However, the definitions can be immediately applied to platoon systems with general IFTs. One advantage of TSS and ATSS is no linearity assumption on platoon systems. Obviously, the definitions can handle type I or II disturbances.

Based on TSS and ATSS, the well-known weak couping theorem was proposed in Swaroop and Hedrick (1996). We will show that the many solutions to combat string instability are specific realizations of the weak coupling theorem.

4.4. LPSS, SLPSS and DSS

To explicitly handle type III disturbances and distinguish the ambiguous definitions, $\mathcal{L}_p$ string stability (LPSS) and strictly $\mathcal{L}_p$ string stability (SLPSS) were proposed in Ploeg,Van De Wouw et al. (2014). The following cascaded state-space system is considered

$$\begin{aligned} &\dot{x}_0 = f_r(x_0, u_r), \\ &\dot{x}_i = f_i(x_i, x_{i-1}), \\ &y_i = h(x_i), \end{aligned} \tag{14}$$

where u_r is the external input of the leading vehicle, x_i is the state for $i \in S_m$, and y_i is the output signal.

Definition 8. (LPSS) The equilibrium point $x_i = 0, i \in S_m$ of system (14) is LPSS if there exist $\mathcal{K}$ functions α and β such that, for any initial disturbance $x(0)$, where $x = (x_0, \cdots, x_m)$ is the state vector, and any control input for the leading vehicle u_r, satisfying

$$\sup_i \|y_i(t)\|_{\mathcal{L}_p} \leq \alpha(\|u_r(t)\|_{\mathcal{L}_p}) + \beta(\|x(0)\|_p), \forall m \in \mathbb{N}. \tag{15}$$

Definition 9. (SLPSS) The equilibrium point $x_i = 0, i \in S_m$ of system (14) is SLPSS if it is LPSS and, in addition, with $x(0) = 0$ it also holds that

$$\|y_i(t) - h(0)\|_{\mathcal{L}_p} \leq \|y_{i-1}(t) - h(0)\|_{\mathcal{L}_p}, \quad \forall i \in S_{e,m}/\{1\}, \forall m \in \mathbb{N}/\{1\}. \tag{16}$$

System (14) specifies the platoon system with the PF topology. The LPSS can be applied for rPF systems immediately. Moreover, the type I, II, and III disturbances can be handled by the LPSS, while SLPSS assumes that the initial condition is zero. The LPSS and SLPSS show huge potential for providing a rigorous basis for often-used string stability definitions for linear systems (Ploeg,Shukla et al., 2014; Ploeg,Van De Wouw et al., 2014).

Note that LPSS and SLPSS only regulate the boundedness of the system outputs but not the convergence. The boundedness is given by the right-hand term of the inequality Eq. (15). Because Eq. (15) holds for any platoon length, the platoon is scalable.

To extend the definitions to systems with type IV disturbances, the disturbance string stability (DSS) was recently proposed in Besselink and Johansson (2017).

Definition 10. (DSS) The equilibrium point $x_i = 0, i \in S_m$ of system (14) is DSS if there exist a class $\mathcal{KL}$ function β, a class $\mathcal{K}$ function σ, and constants $k_1, k_2 \in \mathbb{R}^+$, such that for any initial condition $x_i(0)$ and additive disturbance $\omega_i(t)$, $i \in S_m$, satisfying

$$\sup_i |x_i(0)| < k_1, \sup_i \|\omega_i(t)\|_\infty < k_2, \tag{17}$$

the solution $x_i(t)$, $i \in S_m$, exists for all $t \geq 0$ and satisfies

$$\sup_i |x_i(t)| \leq \beta\left(\sup_i |x_i(0)|, t\right) + \sigma\left(\sup_i \|\omega_i\|_\infty^{[0,t]}\right), \quad \forall m \in \mathbb{N}. \tag{18}$$

The DSS can handle platoon systems with all type I-IV disturbances. As the type IV disturbances are common in real applications, e.g., unmodeled dynamics, DSS is more practical.

Compared with LPSS, the DSS guarantees that the system states reach a region delimited by $\sigma\left(\sup_i \|\omega_i\|_\infty^{[0,t]}\right)$ and has a boundary for any platoon length $m \in \mathbb{N}$, i.e., scalable.

5. Relations of string stability definitions

This section provides major results of relations of above-mentioned definitions. From the perspective of control theory, three types of definitions, i.e., Lyapunov, input-to-output, and input-to-state string stability, are proposed as the bridges of the above-mentioned definitions. All equivalences of definitions are summarized. Based on the equivalences, the proposed input-to-state string stability is recommended as a formal definition of string stability for further related studies.

5.1. Lyapunov string stability

Analogous to Lyapunov stability (Marquez, 2002; Zhou et al., 1996), Lyapunov string stability (LSS) and asymptoticly Lyapunov string stability (ALSS) are proposed.

Definition 11. (LSS) The equilibrium point $x_e = 0$ of a platoon system (7) is said to be LSS if for each $\epsilon > 0, \exists \delta = \delta(\epsilon) > 0$

$$\|x(0)\|_p < \delta \Rightarrow \|x(t)\|_p < \epsilon, \forall t \geq 0, \forall m \in \mathbb{N}. \tag{19}$$

Definition 12. (ALSS) The equilibrium point $x_e = 0$ of a platoon systems (7) is said to be ALSS if it is LSS and $x(t) \to 0$ asymptotically.

Obviously, the LSS and ALSS are straightforward generalization of TSS and ATSS from $\mathcal{L}_\infty$ norm to $\mathcal{L}_p$ norm. It will be technically convenient to further introduce the property for relations of definitions, i.e., the equilibrium point $x_e = 0$ is exponentially stable, denoted as "ES".

5.2. Input-to-output string stability

The input-to-output theory considers a system as a mapping from inputs to outputs and defines stability concerning whether the system output is bounded whenever the input is bounded (Sontag, 2008). For the platoon systems, viewed as a mapping of H: $\omega \to y$, the input-to-output string stability (IOSS) is proposed.

Definition 13. (IOSS) The platoon system (7) is said to be IOSS if all inputs belong to the $\mathcal{L}_p$ space, i.e., $\|\omega\|_{\mathcal{L}_p} < \infty$, and the outputs are once again in the $\mathcal{L}_p$ space for any platoon length $m \in \mathbb{N}$.

Table 2
The additional assumptions on systems.

Definitions	System assumptions
LSS, ALSS, TSS, ATSS	Type I or II
LPSS	Type I, II or III
SLPSS	Type III, PF
IOSS	Type III or IV
FSS, ESS, HTS	Linear, type III, rPLF
SFSS	Linear, type III, PF

A system gain is defined as

$$\gamma_p(H) = \sup \frac{\|y\|_{\mathcal{L}_p}}{\|\omega\|_{\mathcal{L}_p}}. \tag{20}$$

It is immediately obvious that if the platoon system (7) has finite gain for $m \in \mathbb{N}$, then it satisfies IOSS.

5.3. Input-to-state string stability

The definitions of LSS (ALSS) and IOSS are at opposite ends of a spectrum. While LSS applies to equilibrium points of unforced state space realization, i.e., type I or II disturbances, IOSS deals with systems as mappings between inputs and outputs but ignores internal system descriptions, i.e., type III or IV disturbances. The Input-to-State String Stability (ISSS) is introduced to close the gap between these two definitions.

Definition 14. (ISSS) The platoon system (7) is said to be ISSS if there exist a $\mathcal{KL}$ function β, a class $\mathcal{K}$ function γ, and constants $k_1, k_2 \in \mathbb{R}^+$ such that for initial condition disturbances $x(0)$ and $\omega_a(t)$, satisfying

$$\|x(0)\|_p < k_1, \|\omega_a(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]} < k_2, \tag{21}$$

there exists

$$\|x(t)\|_p \leq \beta(\|x(0)\|_p, t) + \gamma(\|\omega_a(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}), \quad \forall t \geq 0, \forall m \in \mathbb{N}. \tag{22}$$

5.4. Relations of definitions

Taking the proposed definitions as bridges, the equivalences of definitions are obtained. The statement "$A \overset{p=2}{\Longrightarrow} B$" denotes "a platoon system is B if it is A with additional assumption $p = 2$". Note when we say "the platoon is A", it means the system (7) has the properties of definition A if it satisfies the additional assumptions of definition A. The additional assumptions on systems are listed in Table 2.

Now we provide the main theorem of definition relations in Theorem 1. We list all relations to our best knowledge even though some of them are straightforward. As shown in Fig. 3, all above-mentioned definitions are classified into three families by their formats, i.e., Lyapunov-like, input-to-state-like, and input-to-output like string stability. Their corresponding commonly used analysis methods are also included, which will be discussed in the next section. Obviously, the proposed ALSS, ISSS, and IOSS are critical for relations among these families.

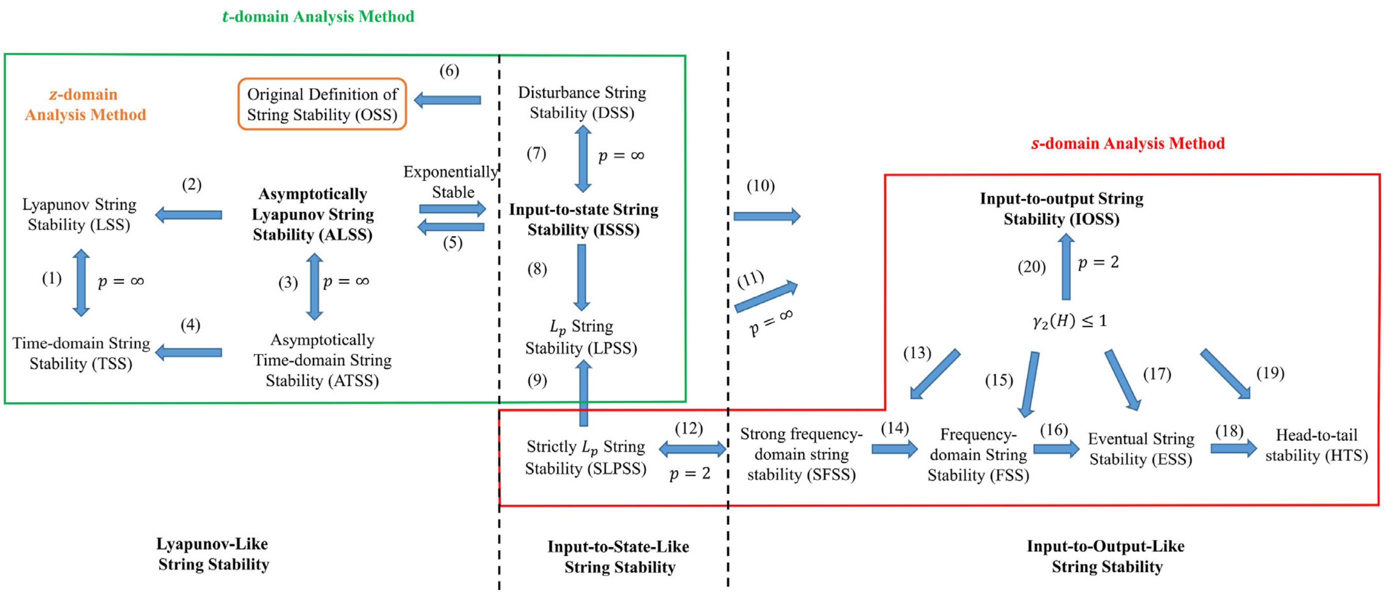

Fig. 3. Relations of string stability definitions and corresponding commonly used analysis methods.

Theorem 1. *For the system (7) under additional assumptions in Table 2, the following equivalences hold:*

(1)	TSS	$\overset{p=\infty}{\Longleftrightarrow}$	LSS;
(2)	$ALSS$	$\overset{p=\infty}{\Longrightarrow}$	LSS;
(3)	$ALSS$	$\overset{p=\infty}{\Longleftrightarrow}$	$ALSS$;
(4)	$ALSS$	$\Longrightarrow$	TSS;
(5)	$ALSS$	$\overset{ES}{\Longrightarrow}$	$ISSS$;
	$ISSS$	$\Longrightarrow$	$ALSS$;
(6)	DSS	$\Longrightarrow$	OSS;
(7)	$ISSS$	$\overset{p=\infty}{\Longleftrightarrow}$	DSS;
(8)	$ISSS$	$\Longrightarrow$	$LPSS$;
(9)	$SLPSS$	$\Longrightarrow$	$LPSS$;
(10)	$ISSS$	$\Longrightarrow$	$IOSS$;
(11)	$LPSS$	$\overset{p=\infty}{\Longrightarrow}$	$IOSS$;
(12)	$SFSS$	$\overset{p=2}{\Longleftrightarrow}$	$SLPSS$;
(13)	$\gamma_2(H)\le 1$	$\Longrightarrow$	$SFSS$;
(14)	$SFSS$	$\Longrightarrow$	FSS;
(15)	$\gamma_2(H)\le 1$	$\Longrightarrow$	FSS;
(16)	FSS	$\Longrightarrow$	ESS;
(17)	$\gamma_2(H)\le 1$	$\Longrightarrow$	ESS;
(18)	ESS	$\Longrightarrow$	HTS;
(19)	$\gamma_2(H)\le 1$	$\Longrightarrow$	HTS;
(20)	$\gamma_2(H)\le 1$	$\overset{p=2}{\Longrightarrow}$	$IOSS$.

Proof. See Appendix B. □

Now we highlight some insightful relations, which may usually be neglected.

Remark 1. The equivalence (12) shows the commonly used definition SFSS is essentially a type of $\mathcal{L}_2$-norm string stability. It means that SFSS considers the energy of disturbances not the boundedness of maximal aptitudes (i.e., $\mathcal{L}_\infty$-norm). As discussed in Caudill and Garrard (1977), Eyre, Yanakiev, and Kanellakopoulos (1998), and Ploeg,Van De Wouw et al. (2014), the $\mathcal{L}_2$ string stability is necessary but insufficient for $\mathcal{L}_\infty$ string stability.

Remark 2. The definitions in frequency-domain, i.e., SFSS, FSS, ESS, and HTS, are particular realizations of $\gamma_p(H) \le 1$ for linear system, $\mathcal{L}_2$ norm, and specific IFTs. Although these definitions are convenient for theoretical analysis and implementations, the limitations deserve more attentions.

Remark 3. The equivalence (5) is critical for understanding the relations between families of Lyapunov and input-to-state like string stability. The right arrow denotes that we can obtain a system's property for all types of disturbances (ISSS) if we know the system's property for type I-II disturbances (ALSS). The major limitation lies in the additional "ES" condition, which is too strict for real platoon systems.

Remark 4. Fig. 3 shows a kind of particularity of ISSS in all definitions. It has relations to all three families of definitions and plays a critical role in the topology. We will analyze the properties of ISSS and provide our recommendation of string stability definition.

5.5. Recommended definition

For a general platoon system (7), we recommend the ISSS as the formal definition of string stability because of the following reasons:

First, the ISSS has the least assumptions on platoon systems. It is suitable for all types of ND, IFT, DC, FG, CQ and disturbances.

Second, the ISSS characterizes three critical properties:

- boundedness of state fluctuations;
- convergence of state fluctuations caused by initial condition disturbances;
- boundedness and convergence hold for any platoon length.

These properties guarantee the scalability of platoon systems, i.e., the addition or removal of vehicles from the platoon will not affect both convergence and boundedness, which is very close to the initial focus of string stability.

Third, the ISSS is convenient for both theoretical analysis and implementations. For platoon control problems, the property in Eq. (22) can be proved by construction of $\mathcal{KL}$ and $\mathcal{K}$ functions, i.e., β and γ, which are independent of the platoon length (see Besselink and Johansson (2017) and Besselink and Knorn (2018) for examples of DSS, i.e., $\mathcal{L}_\infty$-norm ISSS).

For specific platoon systems, e.g., systems suitable for SFSS, FSS, ESS, and HTS, an additional property can be analyzed to further improve the boundedness as

$$\|x(t)\|_p \le \|x(0)\|_p + \|\omega_a(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}, \quad \forall t \ge 0, \forall m \in \mathbb{N}, \tag{23}$$

which denotes the reduction of the state fluctuations. The property is not included in the ISSS because of the concern of its feasibility for general platoon systems.

Therefore, we recommend the ISSS as the formal definition of string stability. For specific problems, sufficient properties can be derived by the property of ISSS and applied for controller design. In this way, the various studies of string stability can be fairly compared via the same definition.

6. Analysis methods

This section introduces the commonly used analysis methods, which derive sufficient conditions of the string stability properties for controller design. To better understand their relations, the analysis methods are classified into three families, i.e., *z*-, *s*-, and time domain analysis methods. Although most methods were designed for specific definitions of string stability and systems, relations of these methods are discussed, which is insightful for future studies.

6.1. z-domain analysis methods

The *z*-domain analysis methods were first proposed to analyze infinite identical linear platoon systems (i.e., linear spatially invariant infinite dimensional systems) (Chang, 1980; Chu, 1974a; 1974b; Liang & Peng, 1999; 2000). The infinity platoons are studied to capture the scalability of the systems, which is critical for string stability properties (Jovanovic & Bamieh, 2005). The essence of the methods is to transform the problem over the vehicle index (instead of discrete time) into *z* domain by bilateral *z*-transform. The platoon systems can be discrete or continuous time systems.

The stability analysis of spatially invariant systems has been studied in control theory for a long time (Curtain, Iftime, & Zwart, 2009; Curtain & Zwart, 2012; Freedman, Falb, & Zames, 1969). An infinite identical platoon system can be described as (Chu, 1974a)

$$\begin{aligned} \dot{x}_k &= \sum_{j=-\infty}^{\infty} (A_{k-j}x_j + B_{k-j}u_j), \\ y_k &= \sum_{j=-\infty}^{\infty} H_{k-j}x_j, \\ u_k &= Fy_k, \end{aligned} \tag{24}$$

where $k \in \mathbb{R}$, and F is a time-invariant feedback gain applied to all identically structured controllers. Matrices A_k and B_k are time-invariant and approach 0 exponentially as $k \to \infty$. By a bilateral *z*-transform and the theorem of convolution (Jury, 1964; Ogata, 1995), we derive

$$\begin{aligned} \dot{\mathcal{X}}(z) &= \mathcal{A}(z)\mathcal{X}(z) + \mathcal{B}(z)\mathcal{U}(z), \\ \mathcal{Y}(z) &= \mathcal{H}(z)\mathcal{X}(z), \\ \mathcal{U}(z) &= F\mathcal{Y}(z), \end{aligned} \tag{25}$$

where *z*-transforms are denoted by corresponding script letters. The closed-loop system dynamics are derived as

$$\dot{\mathcal{X}}(z) = (\mathcal{A}(z) + \mathcal{B}(z)F\mathcal{H}(z))\mathcal{X}(z). \tag{26}$$

The system (26) in *z*-domain can be analyzed by stability analysis methods (Curtain et al., 2010; 2009; Curtain & Zwart, 2012), e.g., eigenvalue analysis methods (Chu, 1974a) and optimization methods (Chu, 1974b; Liang & Peng, 1999). The definition of OSS was commonly used in these studies, as shown in Fig. 3.

The major limitation of *z*-domain methods is that only identical linear platoons can be analyzed. As shown in Eq. (24), the matrices $A_{k-j}, B_{k-j}, H_{k-j}$, and F are only determined by the vehicle index distance, i.e., $k-j$, but independent on the vehicle index k, i.e., the platoon is identical. The linear assumption on platoon systems is also limited for real applications.

Moreover, the infinite case is not always a useful paradigm to understand the long-but-finite case (Curtain et al., 2010). This can be understood intuitively by recognizing that the dynamics of the first and last vehicles may significantly differ from those of other vehicles in a finite-length platoon (Ploeg,Van De Wouw et al., 2014).

6.2. s-domain analysis methods

To better study the characteristics of finite platoons, the *s*-domain analysis methods were proposed and have become the most commonly used methods for string stability issues of linear systems as the theoretical convenience (Jin & Orosz, 2014; 2017; Naus et al., 2010; Ploeg, Scheepers, Van Nunen, Van de Wouw, & Nijmeijer, 2011; Sheikholeslam & Desoer, 1993; Zhang & Orosz, 2016). The critical step of *s*-domain analysis methods is to construct transfer functions between the outputs and inputs of systems by Laplace transform. If the $\mathcal{H}_\infty$ norm of the transfer function is smaller or equal to one, the string stability regarding SFSS (Naus et al., 2010), FSS (Naus et al., 2010), ESS (Khatir & Davidson, 2004), and HTS (Orosz, 2016) can be obtained, as shown in Fig. 3.

The critical step lies at the transfer function analysis of platoon systems for different IFTs. For platoons with *r*PF topology, the characteristic equation of the worst case difference equation was introduced in Swaroop and Hedrick (1999). If propagation dynamics of system states in a string of vehicles can be described as

$$X_i(s) = \sum_{j=1}^{r} G_j(s) X_{i-j}(s), \tag{27}$$

then we can define a polynomial and spectral radius as

$$\begin{aligned} P_r(z) &= z^r - \sum_{1}^{r} \alpha_j z^{r-j}, \\ \rho &= max\{|z| : P_r(z) = 0\}, \end{aligned} \tag{28}$$

where $\alpha_j = \|G_j\|_{\mathcal{H}_\infty}$ and $P_r(z) = 0$ is the characteristic equation. The smaller the value of ρ, the better is the performance of a vehicle string and the tighter is the regulation of intervehicular spacing. The platoon has string stability if it satisfies $\rho < 1$.

For finite platoons with the *r*PF or *r*PLF topology, the link transfer function was introduced in Zhang and Orosz (2016). The evolution of disturbances along network links are measured as follows:

$$Y_i(s) = \sum_{j=1}^{i-1} T_{i,j}(s) Y_j(s), \tag{29}$$

where $T_{i,j}(s)$ is called the link transfer function and describes the influence from $y_j(t)$ to $y_i(t)$. Compared with Eq. (27), Eq. (29) is a generalization of considering the different communication topology for different vehicles.

Both the methods inspire the integration of the transfer function analysis with algebraic graph theory (Biggs, Biggs, & Norman, 1993). If the IFTs can be modeled by the directed graphs, then the closed-loop system dynamics can be represented by compact matrix equations, which provides theoretical convenience for further analysis.

Based on these fundamental ideas, various extensions have been proposed for different scenarios, e.g., time delay (Jin & Orosz, 2014; Liu et al., 2001; Orosz, 2016), packet loss (Teo, Stipanovic, & Tomlin, 2003; Xiao et al., 2008; Xiao et al., 2009), heterogeneous platoons (Shaw & Hedrick, 2007b), range policy (Yanakiev & Kanellakopoulos, 1996; Zhou & Peng, 2005), mixed traffic (Jin & Orosz, 2014; Orosz, 2016), infinite-length platoon (Liang & Peng, 1999).

The first major limitation of s-domain analysis methods is the linear assumptions on systems. Second, as discussed in Theorem 1, s-domain analysis methods regulate the energy of signals but not their maximal amplitudes. Third, systems with generic IFTs (e.g., undirected topology) are still intractable for s-domain analysis methods, although the integration of algebraic graph theory provides a possible direction.

6.3. Time-domain analysis methods

To analyze string stability for nonlinear systems and general IFTs, the Lyapunov techniques and eigenvalue analysis methods have attracted more attentions. To distinguish with the z- and s-domain methods, we call these methods as time-domain methods because they analyze the systems without integral transform.

Lyapunov techniques (Khalil, 1996; Malisoff & Mazenc, 2009; Marquez, 2002) were applied for string stability analysis of nonlinear platoon systems. Generally, the critical step of these methods is to construct Lyapunov functions $V_i(t)$ of interested signals, e.g., system states $x_i(t)$, and analyze the properties of the Lyapunov functions. If all $V_i(t)\to 0$ asymptotically, hence, all $x_i(t)\to 0$ asymptotically, resulting in the string stability property regarding ATSS. In Swaroop and Hedrick (1996), for example, a Lyapunov function $V_i(x)$ was proved existing by the converse Lyapunov theorem. The influence of IFTs is modeled by defining an auxiliary function

$$V(d^{-1},t)=\int_{j=1}^{\infty}V_i(t)d^{-i}. \tag{30}$$

It was proved that if the coupling is sufficiently small, then $V\to 0$ exponentially and hence, $V_i(t)$, $x_i(t)\to 0$ exponentially, which led to the well-known "weak coupling theorem" of string stability.

The major difficulty of Lyapunov techniques lies at the construction of Lyapunov functions, which is a little tricky and sometimes intractable for specific problems. Considering the inherit limitations of z- and s-domain methods, however, Lyapunov techniques receive more attention recently. In a recent work (Besselink & Johansson, 2017), Lyapunov techniques were applied to analyze the string stability regarding DSS. Because of the strong connections between Lyapunov techniques and input-state properties (Sontag & Wang, 1996), Lyapunov techniques deserve more attention for string stability analysis especially for the recommended ISSS definition.

Eigenvalue analysis methods (Barooah & Hespanha, 2005; Cook, 2005; Darbha, Rajagopal et al., 2005; Zheng, Li, Wang et al., 2016) are usually applied for linear platoon systems with generic IFTs. Algeric graph theory (Godsil & Royle, 2013) is introduced to formulate the closed-loop system dynamics. If all the eigenvalues of the closed-loop system dynamics are in the open left half of the complex plane, the system is asymptotically stable. In addition, if the property holds for any platoon length, the system is ALSS. To study this scalability property, infinite-size platoon systems are usually analyzed (Cook, 2005). For generic IFTs, however, because the closed-loop matrix has no special structure, it becomes intolerably difficult to analyze eigenvalues as the size of platoons increases.

To solve this difficulty, the partial differential equation (PDE) method is utilized to estimate the scalability property for BD systems (Barooah et al., 2009; Ghasemi et al., 2013; Hao et al., 2011). The behavior of the least stable eigenvalue is predicted by analyzing the eigenvalues of the PDE method and analyzed to explain the progressive loss of closed-loop stability with increasing size of platoons. However, the PDE method is only suitable for the BD systems, and precision of the behavior prediction of the least stable eigenvalue requires further validation.

6.4. Comparisons of analysis methods

The properties of string stability characterize platoon systems from two perspectives:

- temporal: state fluctuations converge to zero as $t\to\infty$;
- spatial: state fluctuations are bounded as platoon length $m\to\infty$.

As shown in Fig. 4, all above-mentioned analysis methods are proposed to analyze the temporal and spatial properties of systems with their pros and cons:

First, the z-domain methods analyze spatial property only for linear spatially invariant infinite systems, which is rather limited and not always a useful paradigm to understand the long-but-finite case. The temporal property is analyzed by eigenvalue analysis of the closed-loop system dynamics in the z-domain.

Second, the s-domain methods have been extensively utilized benefiting from their theoretical convenience. However, these methods can only analyze temporal properties of linear systems with specific IFTs. Moreover, the system outputs can only be regulated regarding energy ($\mathcal{L}_2$ norm) instead of their maximal amplitudes ($\mathcal{L}_\infty$ norm).

Third, the Lyapunov techniques are proposed for nonlinear systems. Although it is a little tricky for construction of Lyapunov functions, these methods deserve more attentions because of their strong connections with the input-to-state properties, e.g., ISSS.

Fourth, the eigenvalue analysis methods can be used for systems of generic IFTs with the major limitation of linear assumption. The infinite system with generic IFTs is still intractable. The PDE methods are proposed to estimate the scalability property for BD systems.

Now, we recommend possible analysis methods for two existing troublesome issues of string stability analysis. First, the string stability analysis for generic IFTs are still intractable even for the linear platoon systems. The challenging point lies at the spatial complexity which cannot be analyzed by existing methods. The integration of s-domain methods, eigenvalue analysis, and algebraic graph theory is a possible future direction to resolve this problem. Second, the string stability analysis of nonlinear systems is critical because of the nonlinear essence of the vehicle dynamics, difficulty of exact linearizion techniques (e.g., parameter uncertainty (Chehardoli & Ghasemi, 2018; Gao, Hu, Li, Li, & Sun, 2018)), and application of nonlinear controllers. For this issue, the Lyapunov techniques deserve more attention.

7. Alternative properties

This section digs into the commonly used alternative properties of string stability derived by the above-mentioned analysis methods. The properties are compared for the commonly studied platoon system (i.e., linear, identical, and with rPF topology), and their relations are identified. The relations provide insights of the common solutions to combat string instability.

7.1. Studied platoon system

The alternative properties are proposed for specific platoon systems and thus cannot be compared directly. To dig into the relations between various properties, the commonly used platoon systems are selected as linear, identical, and with rPF topology. The system dynamics can be described as

$$\dot{x}_i=h(x_i)=\sum_{j=-r'}^{r}A_j x_{i-j}, \tag{31}$$

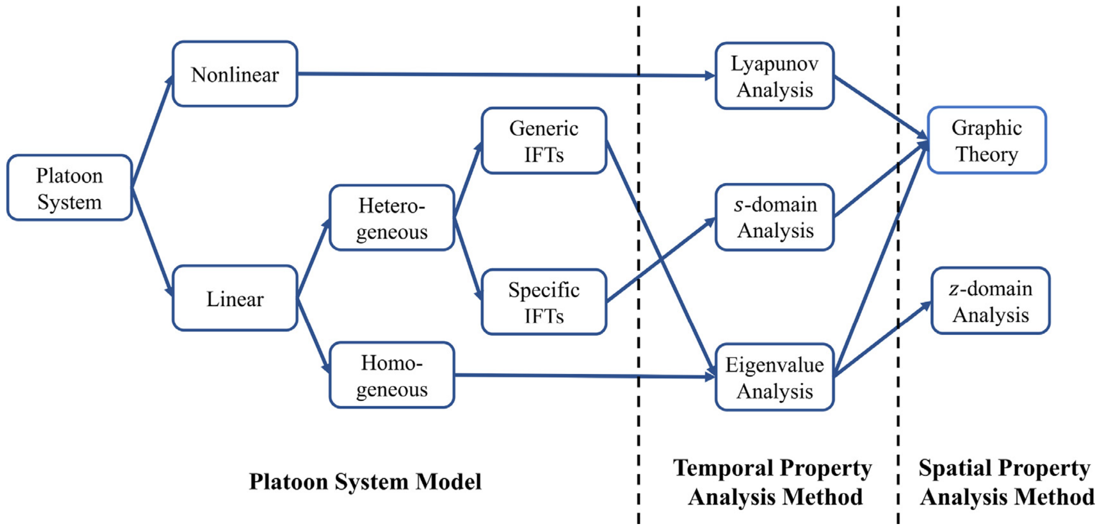

Fig. 4. The practical selection flowchart of analysis methods for different platoon systems.

where $r' = 0, r \in \mathbb{N}$ denotes the rPF topology, and all eigenvalues of A_0 are in the open left-half-plane, i.e., the origin of the unforced system $\dot{x}_i = A_0 x_i$ is exponentially stable. The system can also be considered as a specific case of infinite system as

$$\dot{x}_i = \sum_{j=-\infty}^{\infty} A_j x_{i-j}, \tag{32}$$

where $A_j = 0$ for $j < 0$ or $j > r$.

7.2. Representative alternative properties

We introduce the representative alternative properties derived by time-, z-, and s-domain analysis methods.

First, we introduce the alternative property (P1) derived by z-domain method and eigenvalue analysis in Chu (1974a) as

Property 1. *All the eigenvalues of $\mathcal{A}(z)$ in Eq. (32) have negative real parts for all $|z| = 1$, where $\mathcal{A}(z)$ is the bilateral z-transform of $A_j, -\infty < j < \infty$.*

Second, we introduce the well-known property (P2) in the "weak coupling theorem" derived by the Lyapunov techniques in Swaroop and Hedrick (1996).

Property 2. *For any small $l_j > 0, j = 1, \cdots, r$, the system has the property*

$$\|h(x_i) - h(x_k)\| \le l_0 \|x_i - x_k\| + \sum_{j=1}^{r} l_j \left\| x_{i-j} - x_{k-j} \right\|,$$
$$l_0 > 0, \forall -\infty < i < \infty, \forall -\infty < k < \infty. \tag{33}$$

Third, we introduce some commonly used properties derived by s-domain analysis methods. By Laplace transform of Eq. (31), we derive the transfer function as

$$X_i(s) = \sum_{\substack{j=-r',\\ j\neq 0}}^{r} G_j(s) X_{i-j}(s) = \sum_{j=1}^{r} G_j(s) X_{i-j}(s), \tag{34}$$

where $G_j(s) = A_j/(sI - A_0)$. Then we introduce the properties (P3, P4) in Swaroop and Hedrick (1999) and the property (P5) in Cook (2005) as

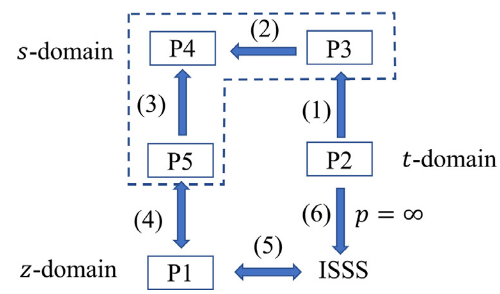

Fig. 5. Relations of the alternative properties for the system (31).

Property 3.

$$\sum_{j=1}^{r} \left\| G_j(s) \right\|_{\mathcal{H}_\infty} < 1. \tag{35}$$

Property 4.

$$\rho < 1, \tag{36}$$

where ρ is defined in Eq. (28).

Property 5. *The equation for z given by*

$$\sum_{\substack{i=-n',\\ i\neq 0}}^{n} G_i(j\omega) z^{-i} = 1, \tag{37}$$

has r roots with $|z| < 1$ and r' roots with $|z| > 1$ for real $\omega \neq 0$.

7.3. Relations of the properties

Now we provide the main theorem of the above-mentioned properties derived by the three families of analysis methods. As shown in Fig. 5, the theorem highlights the relations among. The theorem highlights the relations among these properties and the recommended definition of string stability, i.e., ISSS. Although the theorem is analyzed for the specific system (31), it is helpful for better understanding these properties. The statement "$A \overset{p=\infty}{\Longrightarrow} B$" denotes "the platoon system (31) has the property B if it has the property A with the additional assumption $p = \infty$.

Theorem 2. *For the system (31), the following equivalences of the properties hold:*

(*1*) $P2 \implies P3$;
(*2*) $P3 \implies P4$;
(*3*) $P5 \implies P4$;
(*4*) $P5 \iff P1$;
(*5*) $P1 \iff ISSS$;
(*6*) $P2 \overset{p=\infty}{\implies} ISSS$;

Proof. See Appendix Appendix C. □

Remark 5. The properties in other definitions of frequency-domain, i.e., SFSS, FSS, ESS, and HTS, are special cases of P4 for systems with special IFTs. For examples, for the system (31) with PF topology (i.e., $r = 1$), the P4 is immediately equivalent to SFSS, i.e., the P4 can be directly modified to express the properties in FSS, ESS, and HTS. To make the results concise, we will not discuss these properties.

Remark 6. The "weak coupling" property (P2) shows a strong sufficiency to other properties, i.e., P3 and P4. These properties are essentially a realization of the weak coupling property and provide the quantitative measures of the coupling weakness.

Remark 7. The relations of weak coupling property (P2) shows a trade-off between performance (i.e., convergence) and robustness (i.e., error boundedness). If the subsystems are coupled too weakly that errors of predecessors will not affect following vehicles, the errors will not even be propagated along the string, while, as the cost, the errors cannot converge or converge very slowly.

Remark 8. Various solutions have been proposed to combat string instability for specific systems, e.g., more general information structures (Chien & Ioannou, 1992; Sheikholeslam & Desoer, 1993) and a slightly broader class of spacing policies (Chien & Ioannou, 1992; Eyre et al., 1998; Klinge & Middleton, 2009a; 2009b; Middleton & Braslavsky, 2010; Naus et al., 2010). As implied by Theorem 2, these solutions are essentially realizations of P2.

8. Discussions

8.1. Longitudinal and lateral

Most of the studies mentioned above focus on longitudinal platoon control. For lateral platoon control, lateral string stability was proposed (Alleleijn, Nijmeijer, Öncü, & Ploeg, 2014; Khatir & Davidson, 2005; Papadimitriou & Tomizuka, 2004; Solyom, Idelchi, & Salamah, 2013). A related approach is to simultaneously consider the longitudinal and lateral string stability, which is also known as mesh stability (Pant, Seiler, & Hedrick, 2002).

There are many remaining problems for lateral string stability. Similar to longitudinal string stability, the formal definition of lateral string stability is a critical problem. Theorem 1 can provide insights for this problem, and the ISSS in lateral string stability deserves more attention. The analysis methods of longitudinal string stability can be modified for lateral string stability. For example, Lyapunov technologies have been utilized to obtain the weak coupling theorem for mesh stability (Pant et al., 2002). Their relations discussed in this paper can also be inspiring for lateral string stability.

8.2. Nonlinear systems

Most existing studies of platoon control focus on linear systems. The second and third order linear models are extensively applied. It does not mean the ND is assumed to be linear, but it can be made to appear linear through exact feedback linearization techniques, e.g., Sheikholeslam and Desoer (1993), Stankovic et al. (2000), Ghasemi et al. (2013), and Zheng, Li, Wang et al. (2016). However, the exact feedback linearization techniques require exact prior knowledge of the vehicular dynamics, which is intractable as the existence of parameter uncertainties (Chehardoli & Ghasemi, 2018; Gao et al., 2018). Moreover, the application of nonlinear range policies (Jin & Orosz, 2017) and nonlinear controllers (Santhanakrishnan & Rajamani, 2003) can also make the system nonlinear. The linearization method near the equilibruim points (Jin & Orosz, 2017) suffers the limitation of the region of attraction. Therefore, the string stability analysis for nonlinear systems becomes more critical. As discussed in Section 6, the Lyapunov techniques deserve more attention for this direction.

8.3. General topology

With the development of wireless communication technologies, different information flow topologies (IFTs) have been proposed (Li, Qin et al., 2017; Zhang & Orosz, 2016; Zheng, Li, Wang et al., 2016). The analysis of string stability for general topologies becomes an important problem. As pointed in Section 6, there is no effective analysis method for general IFTs yet. The most related method is to formulate the closed-loop dynamics of the system by graphical theory and analyze the system stability by eigenvalues (Zheng, Li, Li et al., 2016; Zheng, Li, Wang et al., 2016). However, this method can only guarantee convergence and boundedness of a fixed-length platoon. The scalability of the platoon cannot be analyzed by this method, which is most critical for string stability. One possible method is to predict the behaviors of eigenvalues with the increasing of platoon length by approximation methods, which works well for BD topology (Fax & Murray, 2004; Middleton & Braslavsky, 2010; Stüdli, Seron, & Middleton, 2018). For the general IFTs linear systems, the integration of s-domain methods, eigenvalue analysis, and algebraic graph theory is a possible future direction, while for the general IFTs nonlinear systems, the integration of Lyapunov techniques and algebraic graph theory has more potentials.

8.4. General disturbances

Most studies mentioned above only consider the type I-III disturbances, i.e., they assume systems deterministic. However, as known from robust control theory, most real systems have unmodeled dynamics or noises (Sun, Zheng, & Sun, 2018; Zhang, Sun, & Orosz, 2018), and systems can violate the string stability conditions because of the ignored disturbances. To this end, the DSS was proposed in Besselink and Johansson (2017) and the ISSS is proposed and recommended in this paper. To achieve the DSS or ISSS, new controller design methods are required. The data-driven non-model-based method in Gao, Jiang, and Ozbay (2017) and the continuous-time controller design method in Besselink and Johansson (2017) deserve attention in this direction. Moreover, the integration of feedforward and feedback control has the huge potentials to handle the disturbances efficiently and effectively (Mayne, 2016). A recent work Feng, Sun, Zheng, Liu, and Li (2019) proposed a tube-based integrated control method to guarantee the ISSS considering heterogeneous types of disturbances, which deserves more attention for this direction.

8.5. Formation geometry

Formation geometry (FG), e.g., CD, CTH, and NLD, is a critical component of the designed controller for string stability. Because this paper focuses on the definitions and analysis methods, we briefly discuss the studies of FG on string stability. It was shown in Caudill and Garrard (1977) that the CD policy cannot guarantee string stability for identical second-order vehicle dynamics and the PF topology. By introducing more dependence of velocity, the CTH policy was proposed in Chien and Ioannou (1992), which relaxes the coupled dynamics and guarantees the string stability by appropriate choose of parameters (see Remark 8). The minimum boundary of the time headway in the CTH policy has been extensively studied for various situations (Caudill & Garrard, 1977; Middleton & Braslavsky, 2010; Ploeg et al., 2015; Yanakiev & Kanellakopoulos, 1996; Zhou & Peng, 2005). To overcome the traffic inefficiency issue of the CTH policy, the NLD policy was proposed and the corresponding conditions of string stability were analyzed (Orosz, 2016; Yanakiev & Kanellakopoulos, 1995; 1998). The Lyapunov techniques and linearization techniques are usually applied in these studies. As the limitations of the linearization techniques (see Section 8.2), the Lyapunov techniques deserve more attention for this direction. Recently, a delay-based spacing policy was proposed in Besselink and Johansson (2017) to guarantee that all vehicles track the same spatially varying reference velocity profile, as is for example required for heavy-duty vehicles driving over hilly terrain. The string stability properties were also analyzed by the Lyapunov techniques. As the increasing requirements of safety, efficiency, and consumption, more practical and complex FG policies will be proposed. We believe that the Lyapunov techniques will show their huge potentials in this direction.

8.6. Communication quality

The communication quality (CQ), e.g., communication delay and packet loss, affects the system dynamics and related analysis methods. In this subsection, we briefly discuss the related studies on string stability analysis.

The communication delay is usually assumed as fixed or time-varying. It was shown in Mahal (2000) and Liu et al. (2001) that string stability is seriously compromised by fixed communication delay introduced by network when the controllers are triggered by the receipt of either the lead vehicle information or the preceding vehicle information. Theoretical analysis (di Bernardo et al., 2015; Liu et al., 2001; van Nunen, Ploeg, Medina, & Nijmeijer, 2013; Oncu et al., 2012; Peters, Middleton, & Mason, 2014; Xiao et al., 2008; Xiao et al., 2009) and numerical analysis (Jin & Orosz, 2014, 2017; Orosz, 2016) are applied to obtain the boundary requirements of the fixed communication delay for guaranteeing string stability. The major limitation of these studies lies at the negligence of the stochastic property of communication delay. To resolve this problem, time-varying delay was studied, and string stability properties were guaranteed to a given confidence level (di Bernardo et al., 2015; Qin et al., 2017). Unfortunately, the sufficient conditions for string stability are still not available for the stochastic systems. In Fernandes and Nunes (2012), the authors found that using anticipatory information from both the leader and followers could help cancel out the effects of communication delay on string stability if the communication delay had an upper bound.

Packet loss is a source of stochastic communication delay, which is very common for real vehicle platoon. For example, collisions between two or more ongoing transmissions on the wireless medium can render both useless to a receiver, causing packet loss (Guo & Wen, 2016; Lei et al., 2012). The packet loss process can be modeled by discrete-time Markov chains where every state of the chains has its own probability of packet loss (Teo et al., 2002). Moreover, packet loss can be equivalently regarded as random changes in the information flow topology (Moreau, 2005). As shown in Teo et al. (2003), by performing estimation during packet loss, a weak form of string stability can be guaranteed. A control strategy for graceful degradation was proposed in Ploeg et al. (2015), in which the CACC controller degrades to ACC controller in case of persistent packet loss, resulting in a noticeable improvement of string stability characteristics. All these mentioned methods for stochastic communication delay essentially weaken the coupling relations relying on the intervehicle communication, i.e., using anticipatory information (Fernandes & Nunes, 2012), performing estimation (Teo et al., 2003), and graceful degradation Ploeg et al. (2015). As discussed in Remark 8, these solutions are realizations of P2. The trade-off of these methods between the performance and robustness has been generally discussed in Remark 8. Because the stochastic communication delay is equivalent to the random changing IFT, the analytical solutions to this problem are deeply related with those to the problems of general IFTs (see Section 8.3), which requires further efforts.

8.7. Communication mechanism

Most existing methods for vehicular platoon control are based on periodic time-triggered communication mechanism, which places a heavy burden on inter-vehicular communication systems. As reported in Llatser, Festag, and Fettweis (2016), a high message frequency results in a higher number of lost messages and delay due to channel congestion, which has a significant impact on platoon performances. To solve this issue, the communication mechanism of platoon control has become a major concern recently. One hot research topic is the event-triggered communication mechanism (Dolk et al., 2017; Feng et al., 2019; Wen, Guo, Chen, & Gao, 2019), the major idea of which is to only transmit messages after the occurrence of a pre-defined event. Feedback control is designed in Dolk et al. (2017); Wen et al. (2019), where parameters of event-triggered mechanism and controller are fine tuned to guarantee platoon performances, e.g., stability and string stability. During the interval between two events, the controller usually maintains constant until the occurrence of next event. In Feng et al. (2019), an event-triggered feedforward control is designed, which can leverage transmitted messages to plan a sequence of control input instead of a constant. To overcome disturbances between two events, a time-triggered feedback control is integrated to bound systems into a tube near the planned trajectories of systems. The integrated mechanism inherits the communication efficiency of event-triggered communication mechanism as well as the robustness of time-triggered control mechanism, which deserves more attention in this direction.

9. Conclusions

In this paper, we present a literature review of string stability studies from the perspectives of definitions and analysis methods. The relations of ambiguous definitions, analysis methods, and derived alternative properties are discussed rigorously. It provides a solid foundation for understanding the existing studies and resolving future research questions. The representative studies are summarized in Table 3.

Specifically, we outline the commonly used definitions, and propose three types of definitions analogous to stability definitions in control theory. Taking the proposed definitions as bridges, the relations of all these definitions are obtained in Theorem 1. The three critical properties of string stability are summarized as convergence, boundedness, and scalability. Based on these discussions, the definition ISSS is recommended.

Moreover, the major analysis methods and the alternative properties are introduced and compared. From perspectives of temporal and spatial analysis, the essence of these methods is discussed. The derived alternative properties are rigorously discussed for the commonly studied platoon system (linear, identical, and *r*PF) in Theorem 2.

Table 3
A list of some representative studies on string stability.

Papers	ND	Dis	IFT	CQ	FG	DC	Def	Method
Chu (1974a)	2nd-	II	Identical	Ideal	CD	–	ALSS	*z*-
Peppard (1974)	2nd-	III	PF, BD	Ideal	CD	Linear	SFSS	*s*-
Sheikholeslam and Desoer (1990)	Non-	III	PF, PLF	Ideal	CD	Non-	FSS, SFSS	*s*-
Chien and Ioannou (1992)	Non-	III	PF	Ideal	CTH	Non-	SFSS	*s*-
Swaroop and Hedrick (1996)	Non-	II	*r*PF	Ideal	–	–	ATSS	*t*-
Yanakiev and Kanellakopoulos (1996)	2nd-	III	PF, BD	Ideal	CTH	Linear	SFSS	*s*-
Swaroop and Hedrick (1999)	Linear	III	*r*PF	Ideal	CD	Linear	ALSS	*s*-
Liang and Peng (1999)	2nd-	II	Identical	Ideal	CTH	Linear	ALSS	*z*-
Liu et al. (2001)	Non-	III	PLF	Time Delay	CD	Non-	SFSS	*s*-
Seiler et al. (2004)	Linear	II	PF, PLF, BD	Ideal	CD	Linear	FSS, SFSS	*s*-
Shaw and Hedrick (2007a)	Linear	III	PLF	Ideal	CD	Linear	SFSS	*s*-
Naus et al. (2010)	2nd-	III	PF	Time Delay	CTH	Linear	SFSS	*s*-
Fernandes and Nunes (2012)	2nd-	III	PLF	Time Delay	CTH	SMC	–	–
Dunbar and Caveney (2012)	Non-	IV	PLF	Ideal	CTH	MPC	LPSS	*t*-
Ploeg,Shukla et al. (2014)	2nd-	I+III	PF	Ideal	CTH	$\mathcal{H}_\infty$	LPSS, SLPSS	*s*-
Zhang and Orosz (2016)	2nd-	III	*r*PF	Time Delay	Non-	Linear	HTS	*s*-
Dolk et al. (2017)	4nd-	I+III	PF	Time Delay	CTH	Event-	LPSS	*t*-
Jin and Orosz (2017)	2nd-	III	PLF	Time Delay	Non-	Optimal Control	HTS	*s*-
Besselink and Johansson (2017)	Non-	II+IV	PF	Ideal	CTH	Non-	DSS	*t*-
Feng et al. (2019)	3nd-	II+IV	PF	Ideal	CTH	Integrated	ISSS	*t*-

Finally, we discuss the existing challenges and future research directions in this field, e.g., lateral string stability, string stability for general topologies, string stability for general disturbances, and nonlinear systems. We believe that these questions will attract more attention in the near future.

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Appendix A. Tables of Abbreviations

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Table A.1
Table of abbreviations for platoon system description.

Abbreviation	Full name	Abbreviation	Full name
ND	Node dynamics	IFT	Information flow topology
PF	Predecessor following	PLF	Predecessor leader following
BD	Bidirectional	BDL	Bidirectional leader
TPF	Two predecessor following	TPLF	Two predecessor leader following
*r*PF	*r* predecessor following	*r*PLF	*r* predecessor leader following
DC	Distributed controller	MPC	Model Predictive Control
SMC	Sliding mode control	FG	Formation geometry
CD	Constant distance	CTH	Constant time headway
NLD	Nonlinear distance	CQ	Communication quality

Table A.2
Table of abbreviations for string stability definitions.

Abbreviation	Full name	Definition
OSS	Original definition of string stability	1
SFSS	Strong frequency-domain string stability	2
FSS	Frequency-domain string stability	3
ESS	Eventual string stability	4
HTS	Head-to-tail stability	5
TSS	Time-domain string stability	6
ATSS	Asymptotically time-domain string stability	7
LPSS	$\mathcal{L}_p$ string stability	8
SLPSS	Strictly $\mathcal{L}_p$ string stability	9
DSS	Disturbance string stability	10
LSS	Lyapunov string stability	11
ALSS	Asymptoticly Lyapunov string stability	12
IOSS	Input-to-output string stability	13
ISSS	Input-to-state string stability	14

Appendix B. Proof of Theorem 1

Proof. (1) "LSS $\overset{p=\infty}{\Longleftrightarrow}$ TSS": The equivalence is immediately obvious considering the fact

$$\|y(t)\|_\infty = \sup_i |y_i(t)|,$$

by the definitions of LSS and TSS.

(2) "ALSS $\Longrightarrow$ LSS": This is immediately obvious from the definitions of ALSS and LSS.

(3) "ALSS $\overset{p=\infty}{\Longleftrightarrow}$ ATSS": This proof is similar to that of equivalence (1), and so, is omitted.

(4) "ATSS $\Longrightarrow$ TSS": This is immediately obvious from the definitions of ATSS and TSS.

(5) "ALSS $\overset{\text{ES}}{\Longrightarrow}$ ISSS": First, we introduce the lemma below:

Lemma 1. *As proven in Marquez (2002) (Theorem 7.5), assume that the origin of the system (7) is an exponentially stable equilibrium point for the autonomous system $\dot{x} = f(x, 0)$, and the function $f(x, \omega)$ is continuously differentiable and globally Lipschitz in (x, ω). Under these conditions the system is input-to-state stable.*

Benefiting from the similarity between ALSS and Lyapunov stability, ISSS and input-to-state stability, respectively, the system (7) is ISSS if it is ALSS and the origin is exponentially stable (ES), following from Lemma 1.

For "ISSS $\Longrightarrow$ ALSS", let $\omega_a(t) \equiv 0$, i.e., type I or II disturbances, we obtain

$$\|x(t)\|_\infty \le \beta(\|x(0)\|_\infty, t).$$

According to the definition of $\mathcal{KL}$ function, the system is ALSS and the conclusion follows.

(6) "DSS $\Longrightarrow$ OSS": If a system is DSS, for any set of bounded initial disturbances, i.e., there exist a constant $k_1 > 0$ such that $\sup_i |x_i(0)| < k_1$, the state fluctuations of all vehicles remain bounded by $\beta(\sup_i |x_i(0)|, 0)$ and approach zeros as $t \to \infty$ for any platoon length. According to the definition of OSS, the system is OSS.

(7) "ISSS $\overset{p=\infty}{\Longleftrightarrow}$ DSS": At the condition $p=\infty$, the Eq. (22) is

$$\|x(t)\|_\infty \le \beta(\|x(0)\|_\infty, t) + \gamma(\|\omega_a(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}), \quad \forall t \ge 0, \forall m \in \mathbb{N}.$$

Considering the definitions of $\mathcal{L}_\infty$ norm, the conclusion follows.

(8) "ISSS $\Longrightarrow$ LPSS": If the system is ISSS, we obtain

$$\|x(t)\|_p \le \beta(\|x(0)\|_p, t) + \gamma(\|u_r(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}), \quad \forall t \ge 0, \forall m \in \mathbb{N},$$

where u_r is considered as ω_a in LPSS. Because $y=g(x)$ is global Lipschitz, there exists a constant $L\in\mathbb{R}^+$ such that $\forall t \ge 0$,

$$\|y(t)\|_p \le L\|x(t)\|_p \le L\beta(\|x(0)\|_p, t) + L\gamma(\|u_r(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}), \forall t \ge 0, \forall m \in \mathbb{N}. \tag{B.1}$$

For $\mathcal{L}_\infty$ norm, there exists a class $\mathcal{K}$ function γ_1, γ_2 such that

$$\sup_i \|y_i(t)\|_{\mathcal{L}_\infty} = \|y(t)\|_{\mathcal{L}_\infty} \le \gamma_1(\|x(0)\|_\infty) + \gamma_2(\|u_r(\cdot)\|_{\mathcal{L}_\infty}), \forall m \in \mathbb{N},$$

and the conclusion follows.

(9) "SLPSS $\Longrightarrow$ LPSS": It is immediately obvious by the definitions of SLPSS and LPSS.

(10) "ISSS $\overset{p=\infty}{\Longrightarrow}$ IOSS": If the system is ISSS, when $x(0)=0$, we obtain

$$\|x(t)\|_p \le \gamma(\|w_a(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}), \forall t \ge 0,$$

for all $w_a \in \mathcal{L}_\infty$. Then, similarly to Eq. (B.1), for $\mathcal{L}_\infty$ norm, there exists a constant $L\in\mathbb{R}^+$ such that

$$\|y(t)\|_{\mathcal{L}_\infty} \le L\|x(t)\|_{\mathcal{L}_\infty} \le L\gamma(\|w_a(\cdot)\|_{\mathcal{L}_\infty}^{[0,t]}) < \infty.$$

It means that $y(t)\in\mathcal{L}_\infty$ and the system is IOSS.

(11) "LPSS $\overset{p=\infty}{\Longrightarrow}$ IOSS": If the system is LPSS, for $p=\infty$ we obtain

$$\sup_i \|y_i(t)\|_{\mathcal{L}_\infty} \le \alpha(\|u_r(t)\|_{\mathcal{L}_\infty}) + \beta(\|x(0)\|_\infty) < \infty, \quad \forall m \in \mathbb{N}.$$

Therefore, $y(t)\in\mathcal{L}_\infty$ and the conclusion follows.

(12) "SFSS $\overset{p=2}{\Longleftrightarrow}$ SLPSS": Let $G_{i-1,i}(s)$ denote the transfer function of the outputs for vehicles $i-1$ and i. Then we have $Y_i(s) = G_{i-1,i}(s)Y_{i-1}(s)$, where $Y_i(s)$ is the Laplace transform of $y_i(t)$. Since the $\|\cdot\|_\infty$ is induced by the $\mathcal{L}_2$ norms of the respective signals (see Zhou et al. (1996) for details), we obtain

$$\|G_{i-1,i}\|_{\mathcal{H}_\infty} = \sup_{y_{i-1}\in\mathcal{L}_2} \frac{\|y_i\|_{\mathcal{L}_2}}{\|y_{i-1}\|_{\mathcal{L}_2}}.$$

Note that according to Parseval's theorem, the time-domain and frequency-domain $\mathcal{L}_2$ norms are equal (Zhou et al., 1996). Then, we obtain

$$\sup_i \|G_{i-1,i}\|_{\mathcal{H}_\infty} \le 1 \Longleftrightarrow \|y_i\|_{\mathcal{L}_2} \le \|y_{i-1}\|_{\mathcal{L}_2}, \forall i \in S_{e,m}$$

and the equivalence follows.

(13) "$\gamma_2(H)\le 1 \Longrightarrow$ SFSS": If the system is not SFSS, i.e., there exists a $i\in S_{e,m}$ such that $\left\|G_{i-1,i}(j\omega)\right\|_{\mathcal{H}_\infty} > 1$, let $\omega_j(t)=0, \forall j\neq i$, for the disturbances $\omega(t)$. Then we have $\|\omega(t)\|_{\mathcal{L}_2} = \|\omega_i(t)\|_{\mathcal{L}_2}$. The system gain is obtained by

$$\begin{aligned}\gamma_2(H) &= \sup_{\omega(t)\in\mathcal{L}_2} \frac{\|y(t)\|_{\mathcal{L}_2}}{\|\omega(t)\|_{\mathcal{L}_2}}, \\ &= \sup_{\omega_i(t)\in\mathcal{L}_2} \frac{\|y(t)\|_{\mathcal{L}_2}}{\|\omega_i(t)\|_{\mathcal{L}_2}}, \\ &\ge \sup_{\omega_i(t)\in\mathcal{L}_2} \frac{\|y_i(t)\|_{\mathcal{L}_2}}{\|\omega_i(t)\|_{\mathcal{L}_2}}, \\ &= \|G_{i-1,i}(j\omega)\|_{\mathcal{H}_\infty} \\ &> 1,\end{aligned}$$

which is in contradiction to $\gamma_2(H)\le 1$. The conclusion follows.

(14) "SFSS $\Longrightarrow$ FSS": It is immediately obvious from the definitions of SFSS and FSS.

(15) "$\gamma_2(H)\le 1 \Longrightarrow$ FSS": The conclusion follows the equivalences (13) and (14) as "$\gamma_2(H)\le 1 \Rightarrow$ SFSS $\Rightarrow$ FSS".

(16) "FSS $\Longrightarrow$ ESS": It is immediately obvious from the definitions.

(17) "$\gamma_2(H)\le 1 \Longrightarrow$ ESS": It can be proved by the equivalences of (15) and (16).

(18) "ESS $\Longrightarrow$ HTS": It is immediately obvious from the definitions.

(19) "$\gamma_2(H)\le 1 \Longrightarrow$ HTS": The conclusion follows the equivalences (15), (16) and (18) as "$\gamma_2(H)\le 1 \Rightarrow$ FSS $\Rightarrow$ ESS $\Rightarrow$ HTS".

(20) "$\gamma_2(H)\le 1 \overset{p=2}{\Longleftrightarrow}$ IOSS": If $\gamma_2(H)\le 1$, then the system gain is bounded by $\mathcal{L}_2$ norm, and so, the system is IOSS with $p=2$. □

Appendix C. Proof of Theorem 2

Proof. (1) "P2 $\Longrightarrow$ P3": The system (31) has

$$\|h(x_i) - h(x_k)\| = \left\| \sum_{j=1}^r A_j(x_{i-j} - x_{k-j}) \right\| \le \sum_{j=1}^r \left\|A_j\right\| \left\|x_{i-j} - x_{k-j}\right\|.$$

The P2 characterizes that $\left\|A_j\right\|, j=1,\cdots,r$ are sufficiently small. Because $A_j, j=1,\cdots,r$ denote the coupling of interconnected subsystems, the related theorem was called as "weak coupling theorem."

If $\left\|A_j\right\|, j=1,\cdots,r$ are sufficiently small such as

$$\sum_{j=1}^r \left\|A_j\right\| \le \frac{1}{\|(sI-A_0)^{-1}\|_{\mathcal{H}_\infty}},$$

then, by the property of $\mathcal{H}_\infty$, we obtain

$$\begin{aligned}\sum_{j=1}^n \left\|(sI-A_0)^{-1}A_j\right\|_{\mathcal{H}_\infty} &\le \sum_{j=1}^n \left\|(sI-A_0)^{-1}\right\|_{\mathcal{H}_\infty} \left\|A_j\right\| \\ &\le \left\|(sI-A_0)^{-1}\right\|_{\mathcal{H}_\infty} \sum_{j=1}^n \left\|A_j\right\| \\ &\le 1,\end{aligned}$$

i.e., the system has the property P3.

(2) "P3 $\Longrightarrow$ P4": It has been proved in Swaroop and Hedrick (1999) (Proposition 1).

(3) "P5 $\Longrightarrow$ P4": By P5, the Eq. (37) has r roots with $|z|<1$ and 0 roots with $|z|>1$ for all real $\omega\neq 0$. By definition in Eq. (28), P4 characterizes the worst case of P5 regarding ω. Because P5 holds for all real $\omega\neq 0$, P4 holds if P5 holds.

(4) "P5 $\Longleftrightarrow$ P1": To prove the theorem, we first introduce a lemma proven in Cook (2005). For the system (31), we introduce a discrete Fourier transform of the infinite-dimensional state-vector by defining

$$\tilde{x}(\theta, t) = \sum_i x_i(t)\exp(-ji\theta),$$

so that

$$\dot{\tilde{x}}(\theta, t) = F(\theta)\tilde{y}(\theta, t),$$

where

$$F(\theta) = \sum_{r=-n'}^{n} A_r \exp(-ji\theta). \tag{C.1}$$

Then we introduce the lemma as below (see Cook (2005) Theorem 2).

Lemma 2. *For system (31), all nonzero eigenvalues of $F(\theta)$ in Eq. (C.1) are in the open left half of the plane for all real θ if and only if the P5 holds.*

Note that the z transform in P1 with $|z| = 1$ is equivalent to the discrete Fourier transform in Eq. (C.1). By Lemma 2, therefore, the P1 holds if and only if P5 holds is true.

(5) "P1 $\iff$ ISSS": Be the definition of ALSS, the system (31) has the property P1 if and only if it is ALSS. Because the origin of the unforced system is exponentially stable (see Eq. (31)), the system is ISSS according to Theorem 1 and vice verse.

(6) "P2 $\implies$ ISSS": By the "weak coupling theorem" in Swaroop and Hedrick (1996), the system (31) is ATSS is the P2 holds. According to Theorem 1, the system is ALSS if and only if it is ATSS for $p = \infty$. Because the origin of the unforced system is exponentially stable, the system is ISSS for $p = \infty$ according to Theorem 1. □

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